CMSC427/828E Spring 2000
Homework # 4

March 8, 2000

1. First state the equations of Q_1, Q_2 and R :

$$\begin{aligned} Q_1 &= P_1 + t(P_2 - P_1) & (1)\\ Q_2 &= P_2 + t(P_3 - P_2) & (2)\\ R &= Q_1 + t(Q_2 - Q_1) & (3) \end{aligned}$$

Now, substitute Q_1, Q_2 in equation 3 by equations 1 and 2

$$\begin{aligned} R &= P_1 + t(P_2 - P_1) + t(P_2 + t(P_3 - P_2) - P_1 - t(P_2 - P_1)) \\ &= P_1 + t(P_2 - P_1) + t(P_2 - P_1) + t^2(P_3 - P_2 - P_2 + P_1) \\ &= P_1 + 2t(P_2 - P_1) + t^2(P_3 - 2P_2 + P_1) & (4) \end{aligned}$$

The outline of R is a parabola as in fig 1.

2. In this question, we want map the point (x, y) to (x_S, y_S) as in figure 2:

Looking on the x-axis, we will notice that $\frac{c}{d} = \frac{a}{b}$, and thus :

$$\frac{(x_S - u_{min})}{(u_{max} - u_{min})} = \frac{(x - x_{min})}{(x_{max} - x_{min})} \quad (5)$$

multiply by $(u_{max} - u_{min})$, and getting $u_m in$ to the right hand side results in :

$$x_S = u_{min} + \frac{(u_{max} - u_{min})}{(x_{max} - x_{min})}(x - x_{min}) \quad (6)$$

and similarly, we can derive y_S, as :

$$y_S = v_{min} + \frac{(v_{max} - v_{min})}{(y_{max} - y_{min})}(y - y_{min}) \quad (7)$$

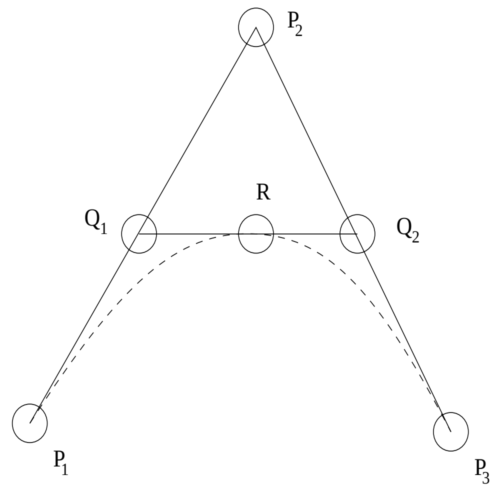

Figure 1: Outline for solution

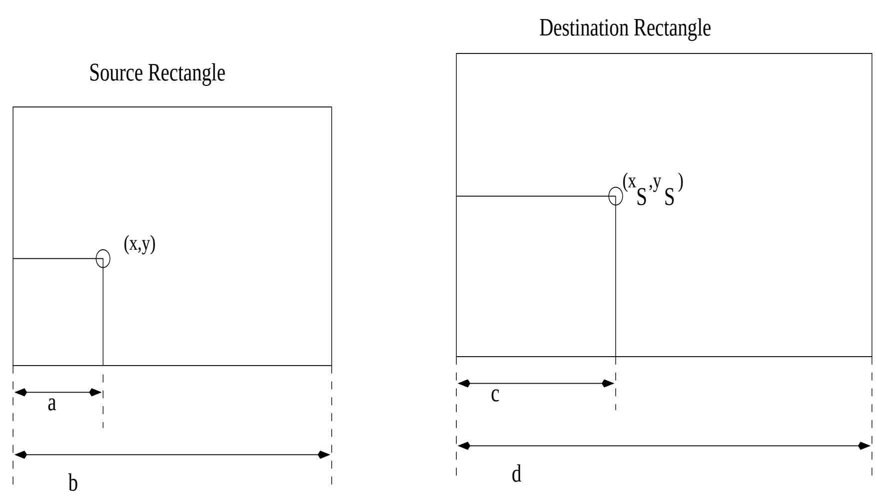

Figure 2: Mapping question

3. Suppose that the polygon consists of n points, where $n > 3$, so the following algorithm can detect whether the polygon is convex or not.

   Algorithm

   for $(i = 1; i \leq n; i++)\{$
       draw line from point i to mod(i+2,n);
       check if the midpoint of line $(i, i+2)$ lies within the polygon;
       if not then return the polygon is not convex and exit;
   }
   The polygon is convex;

4. (a) Yes. The vector $k = -i$ satisfies the two equations.
   (b) No.
   (c) In the 3D, the answer to both parts is Yes, where $k = i \times j$.

5. We know that cross reference equation

$$u\ timesv = \begin{bmatrix} u_2v_3 - u_3v_2 \\ -(u_1v_3 - u_3v_1) \\ u_1v_2 - u_2v_1 \end{bmatrix} \tag{8}$$

   By applying the previous equation, we can reach (1,2,3) by substituting u by e_1 and v by e_2, and the answer will be e_3. The $(3, 2, 1)$ can be proved in the same way.

   The previous part depends on mathematics. If we apply the right-hand-rule to $(1, 2, 3)$, we will reach $e_1 \times e_2 = e_3$. Whereas, applying the left-hand-rule to $(3, 2, 1)$ results in $e_3 \times e_2 = e_1$.

6. n is the vector orthogonal to the plane consists of the three points p_1, p_2, p_3 (assume these points are no collinear).

   We know that n is perpendicular to plane (p_1, p_2, p_3), and thus is perpendicular to any line in this plane. Since, line $(p - p_1)$ belongs to the plane, then n is orthogonal to $(p - p_1)$. So, the following equation is satisfied:

$$n \cdot (p - p_1) = 0 \tag{9}$$