Practice Sheet for CMSC 828J Midterm
Spring 2009

Topics covered
1. Template Matching
   o Distance Transform
   o Chamfer matching
   o RANSAC
   o Hough Transform
   o Interpretation Tree
   o Transformation Space methods
2. 3D Geometry
   o Matrix representation of translation and rotation
   o Perspective and scaled orthographic projection
   o Affine transformations
   o Projective transformations
   o Invariants
   o Aspect graphs
3. Linear supspaces
   o PCA
   o LDA
4. Lighting
   o Image normalization, normalized correlation, direction of image gradient.
   o Lambertian reflectance
   o 3D linear representation of images with no shadows.
   o 9D representation with attached shadows.
5. Image Spaces
   o Kendall's shape space – basic idea
   o Procrustes distance
   o Definition of manifold, geodesics
6. Deformable Objects
   o Use of dynamic programming for curve matching
   o Medial axis, grassfire algorithm.
   o Thin-plate splines

I am allowed to base an extra credit challenge problem on material in one of the readings.

Practice Problems

These problems are meant to give you examples of some things that might be asked on the midterm. It should be helpful to do these while studying. These are NOT comprehensive, and the midterm may include questions on any of the above topics.

1. **Template Matching**
   a. Suppose a 5x5 image has edge pixels at I(1,2) and I(3,4). Draw the image and its distance transform, using the Manhattan distance.
   b. Suppose you want to find sets of points that form circles using the Hough transform. Describe a circle space, and the set of buckets you would fill in that space for a point located at (3,3).
2. **Geometry**
   a. Give an example of a 3x3 rotation matrix.
   b. Give an example of a 3x3 rotation matrix that will rotate a point on the x axis so that it is transformed to lie on the y axis.
   c. Consider affine transformations, written as $Ax + t$, where x is a point, A is a 2x2 matrix, and t is a translation. Suppose we restrict ourselves to affine transformations in which A has a determinant of 1. Describe an image property of three points that is invariant to such a transformation.
   d. Consider the 2D projective transformation:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

   Now, consider applying this projective transformation to a point located in the plane, with coordinates (1,0). First, what are the homogenous coordinates of this point? Next, where will this point appear after the projective transformation? Where will a point with coordinates (10,0) appear after applying this transformation?
   e. Now, consider applying this transformation to the line $y=0$. Where will this line appear after this transformation? What about $y=1$? Where will these two lines intersect, after being transformed?
3. **Linear Subspaces**
   a. Suppose we have two classes, each class containing a set of points. We run LDA and PCA to produce a 1D linear subspace to approximate the points. We find that LDA and PCA produce the same 1D subspace. Give an example of a set of points that would produce this result.
   b. Can you give a general, complete description of point sets that would produce this result (don't just repeat the definitions of PCA and LDA. Is there a more concise description of the point sets that can produce this?)
4. **Lighting**
   a. Consider a 2D world, in which all surface normals have no (or zero) z component, and all lights have no z component. What would be a good low-dimensional approximation to the set of images that a circle could

produce, allowing for attached shadows? (That is, sketch how you would derive the spherical harmonics results for a simpler, 2D world).

5. **Image Spaces**
    a. Find the procrustes distance between the two point sets: {(2,3), (6,6), (5,7)} and {(7,6), (12,6), (7,1)}. For this problem, you should account for translation by using the first point as the origin. (That is, do not translate the points so the center of mass is at the origin, but rather so that the first point is at the origin.
    b. Suppose I define a local tangent space on the 2D plane by assuming that the cost of moving a short distance in any direction, at the location (x,y) is proportional to:

$$\frac{1}{1+\sqrt{x^2 + y^2}}$$

Equipped with this distance, the points on the 2D plane form a manifold. Give an example of a geodesic on this manifold.

6. What does the grassfire algorithm produce as a skeleton for an arbitrary regular polygon?