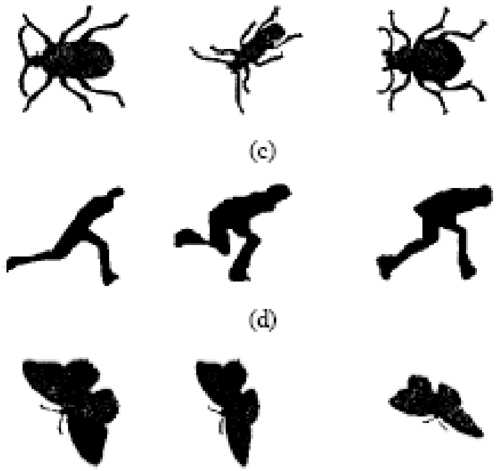
Silhouettes

Silhouette Comparison

- Many Applications
    - CBIR, logos, handwriting, human pose recognition, plant species identification.
- Shapes undergo many transformations that can be hard to model
    - Articulations
    - Deformations
    - Within-class variation



Classic approaches

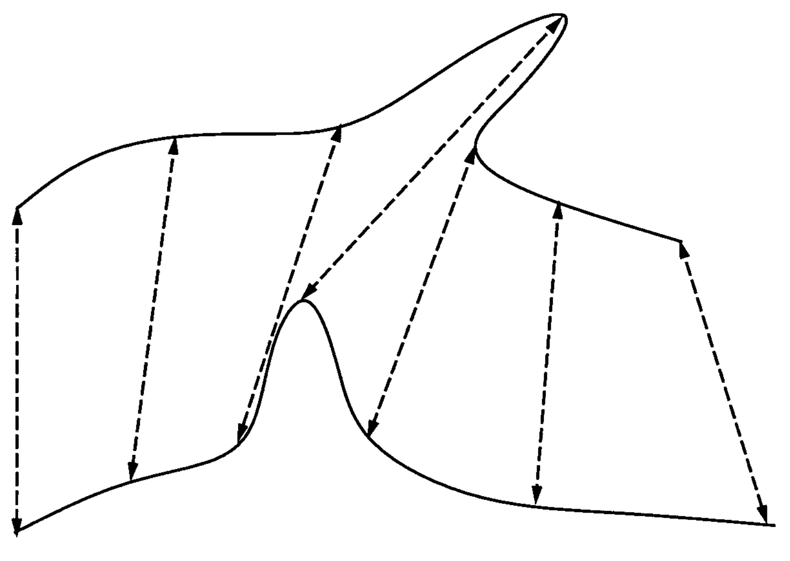
- Moment-based comparison
- Fourier descriptors
- Syntactic Pattern Recognition

Matching 1D contour

- Silhouette boundary is 1D contour
- Points on boundary are ordered.
  – This allows matching with Dynamic programming.
    - Order preserving matching
    - Many-to-one, allows “stretching”.
    - Efficient, globally optimal solution.

Example of DP contour matching

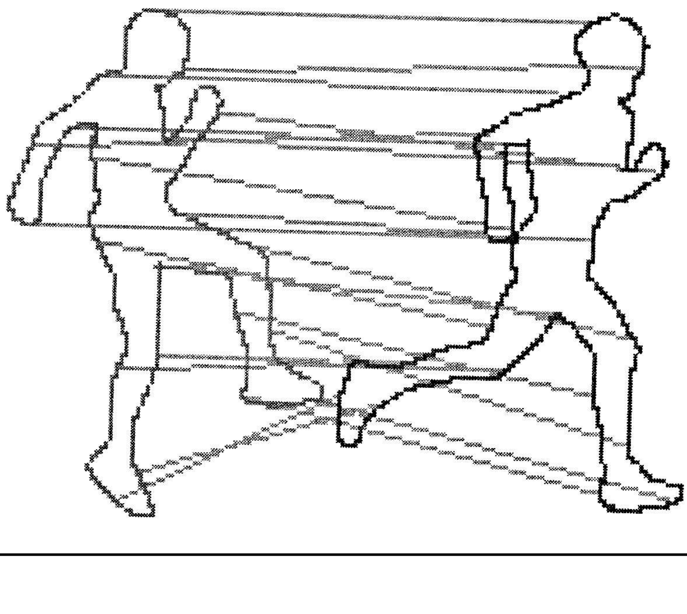
- Select points on boundary (uniform sampling, curvature extrema, …)
- Describe point (curvature, shape context…)
- Define a distance between two points and their descriptions (difference in curvature, Chi-square distance on histograms, …)
- Define a cost for leaving a point unmatched (optional)
- Find the best, order preserving many-to-one matching between points



Algorithm

- Match contour points (p1, …pn), & (q1, … qm).
- Assume a starting match (either get one heuristically, or try many.
- Build table of costs reflecting all possible matches.
  - c(pi,pj) is the cost of matching pi to qj.
  - o(pi) is the cost of not matching pi.
  - C(i,j) is the cost of the best matching that accounts for (p1…pi), (q1, … qj).
  - C(1,1) = c(1,1).
  - C(i,j) = min (C(i-1,j-1)+c(i,j), C(i-1, j)+min(o(pi), c(pi,qj), C(i,j-1)+min(o(pj), c(pi,pj)).
- C(n,m) is the cost of matching entire contours.
  - Can also keep track of which points matched.

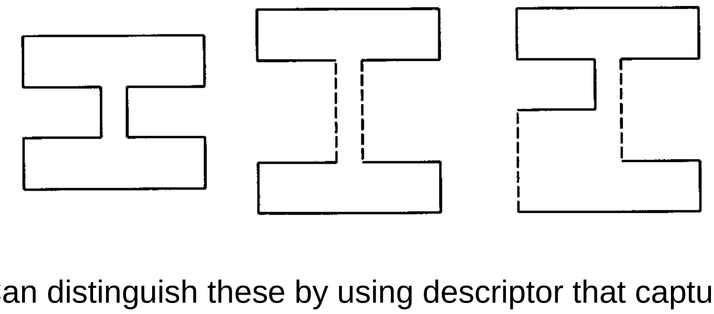
Example



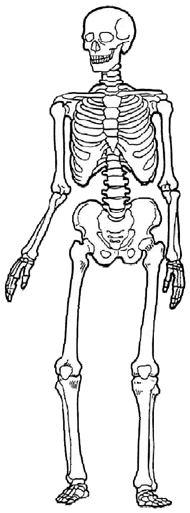
Additional possibilities

- Add cost for amount of stretching
  - Cost for amount of deformation.
  - Requires each step to move forward on both curves, so amount of stretching is available to cost function
- Use shortest path algorithm instead of DP.
- Hierarchical matching
  - Match midpoints. Cost is similarity of triangles formed by end and midpoints, plus cost of matching the two halves of the contours, separated by midpoints.

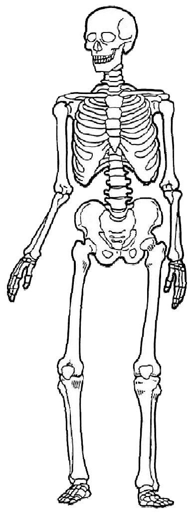
1D or 2D similarity?



Can distinguish these by using descriptor that captures non-local portions of curve.



Skeletons



By Kenny Weiss,
Ryan Farrell

CMSC 828J – Approaches to Representing and Recognizing Objects
Dr. David Jacobs, Spring 2006

What is a Skeleton?

Shapes are normally described by their boundaries. Here we shift the description to the interior by using as primitive, a growth or a disc (growth of a point). Two-dimensional, filled-in shapes are then described using the discs which just fit inside. The description consists of two parts: **the symmetric axis or locus of centers** of these discs and **the radii of these discs** along the symmetric axis.

- Harry Blum [HB]

Examples (Blum)

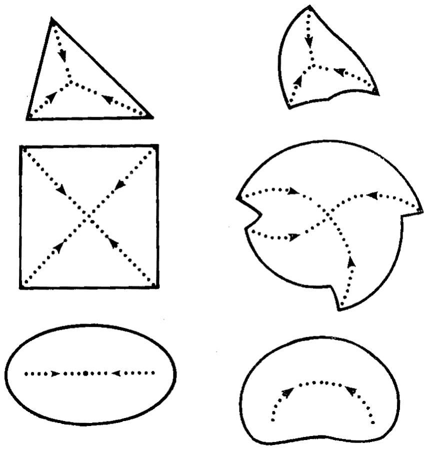
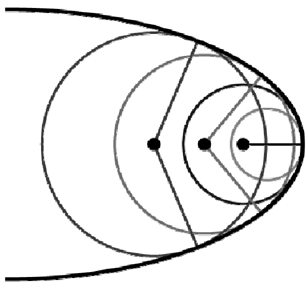


Image Credit: H. Blum [HB, p. 239]

- Symmetric axis (sym-axis) indicating direction of increasing Symmetric Distance (sym-dist)
- Both interior and exterior sym-axes in general

Symmetric/Medial Axis, Skeleton

- Locus of centers of “maximal discs”
- Locus of centers of circles tangent to curve in two or more places



POP QUIZ

What object
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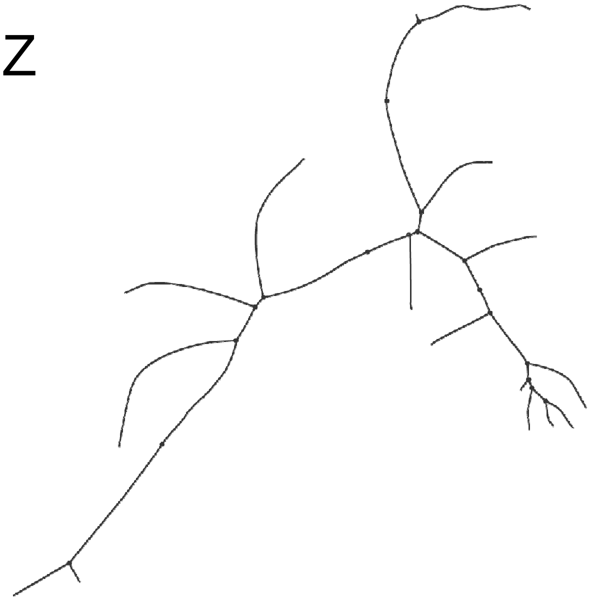
skeleton
represent?



HINT

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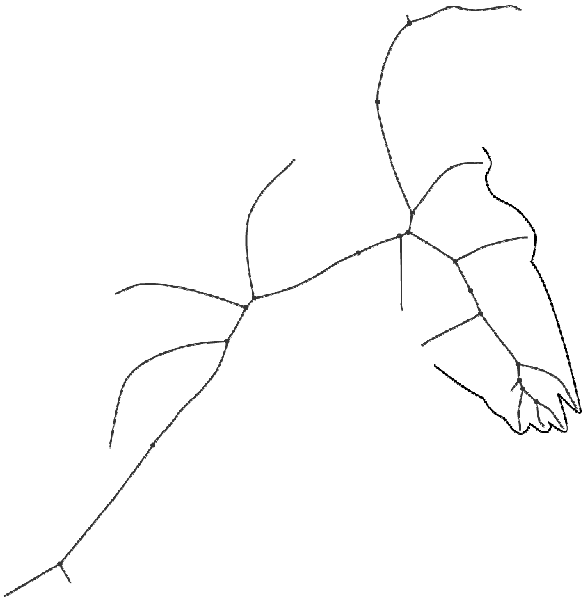

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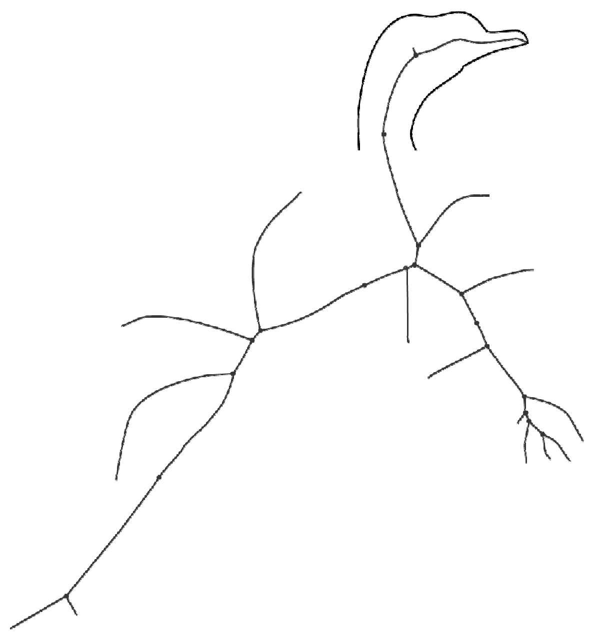
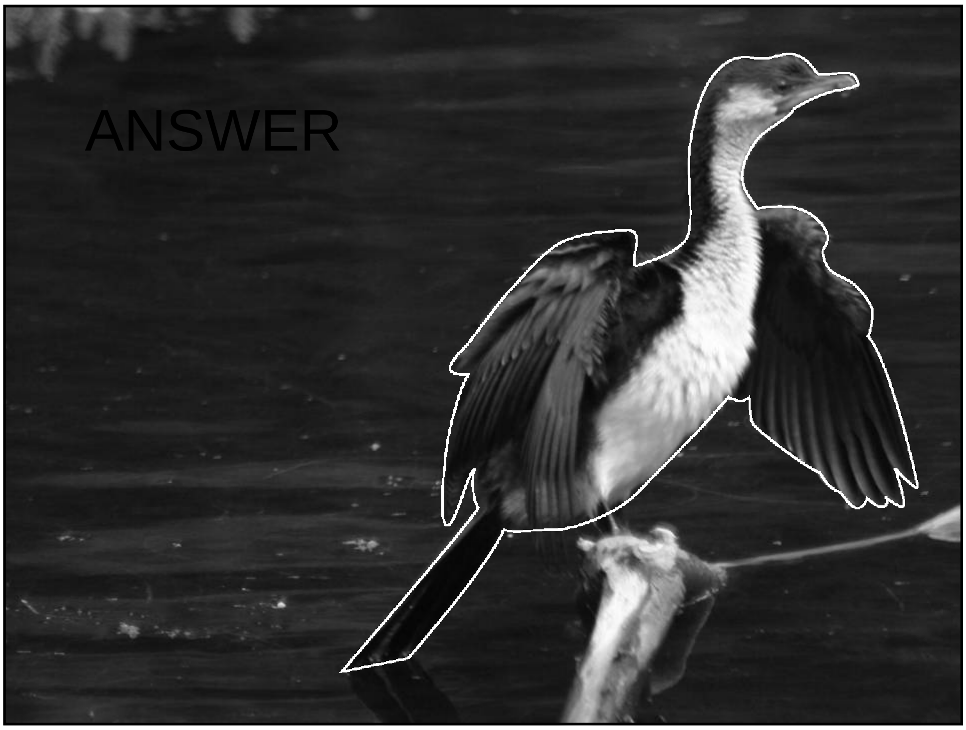
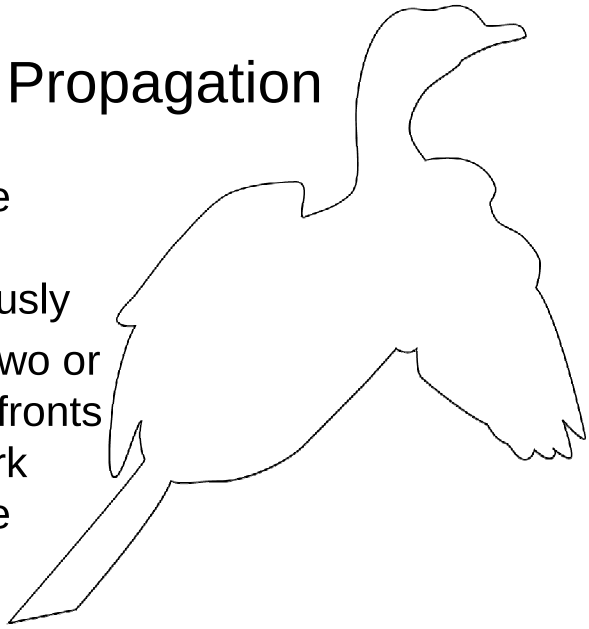




Grassfire Propagation

- Ignite entire perimeter simultaneously
- Wherever two or more wavefronts collide, mark point on the skeleton



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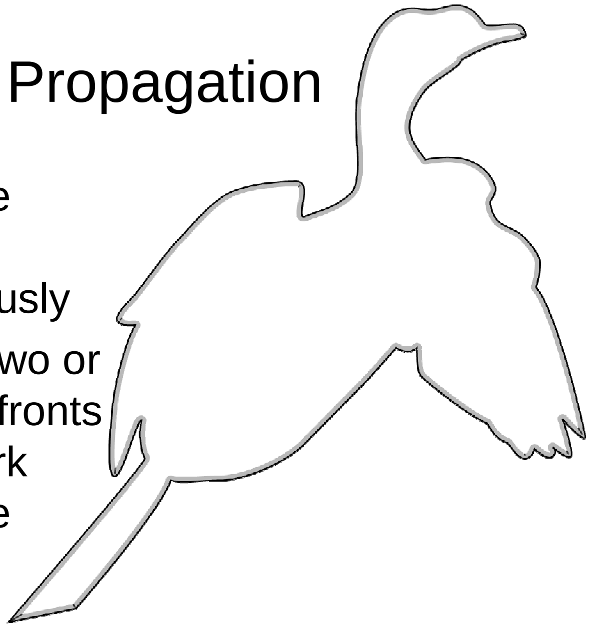

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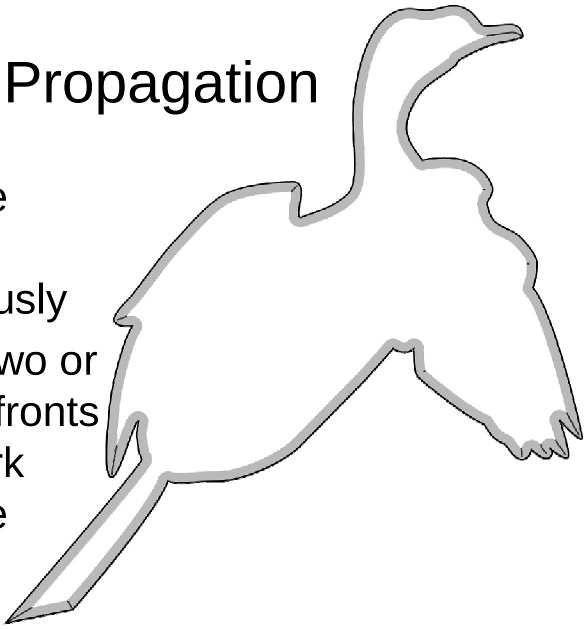

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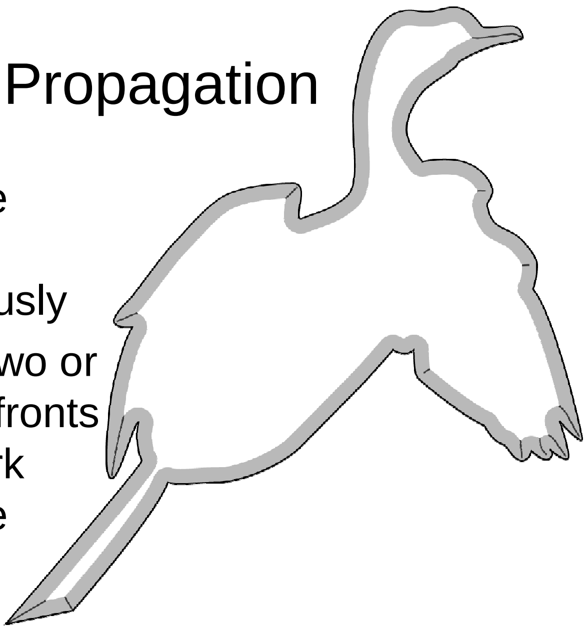

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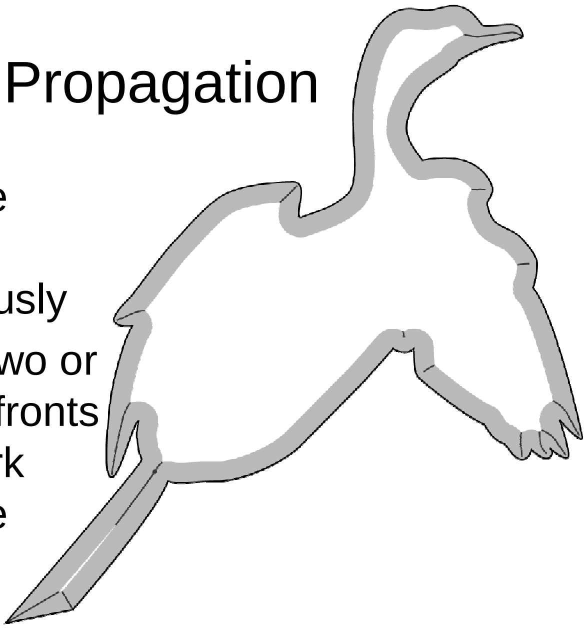

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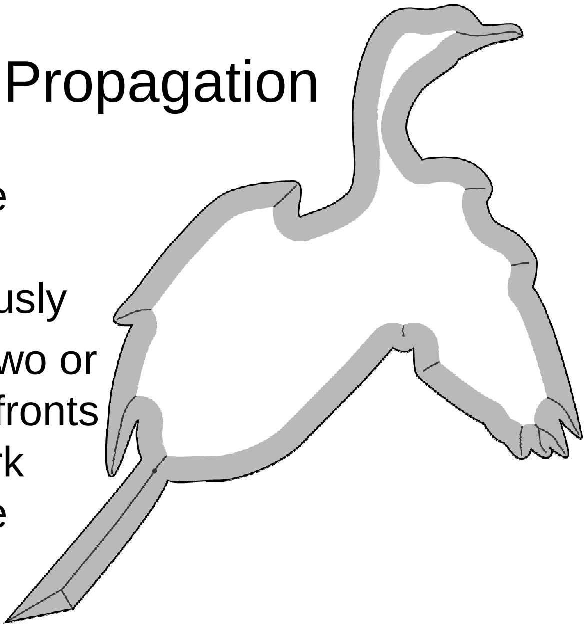

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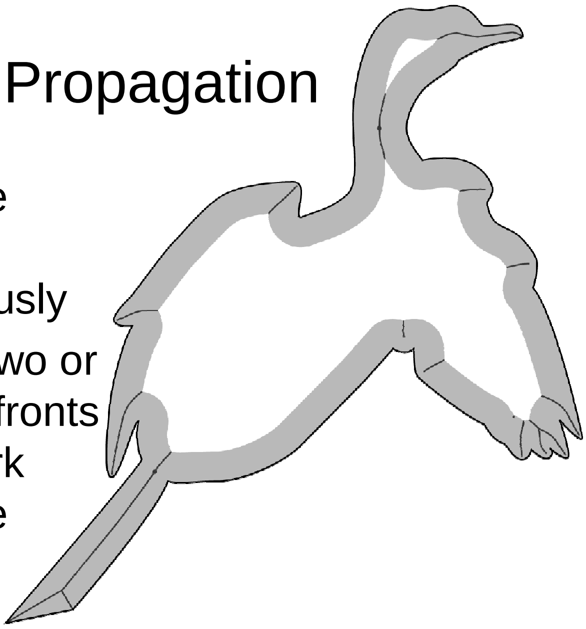

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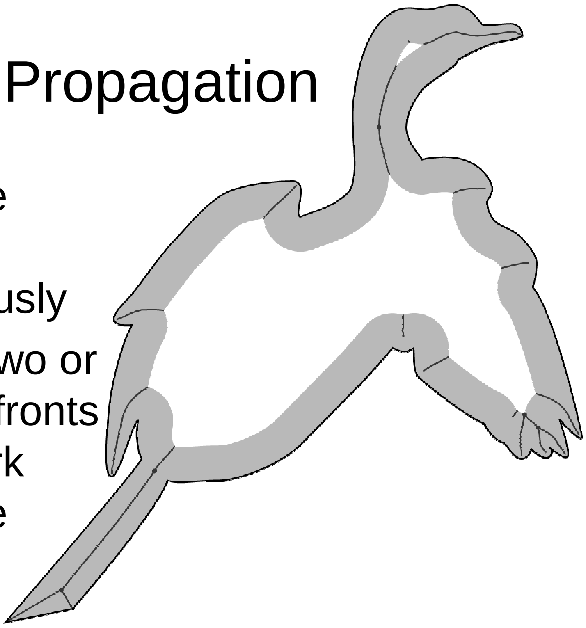

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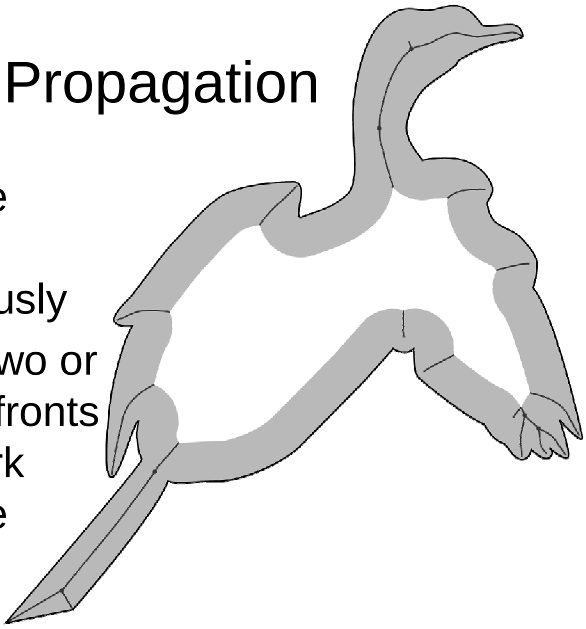

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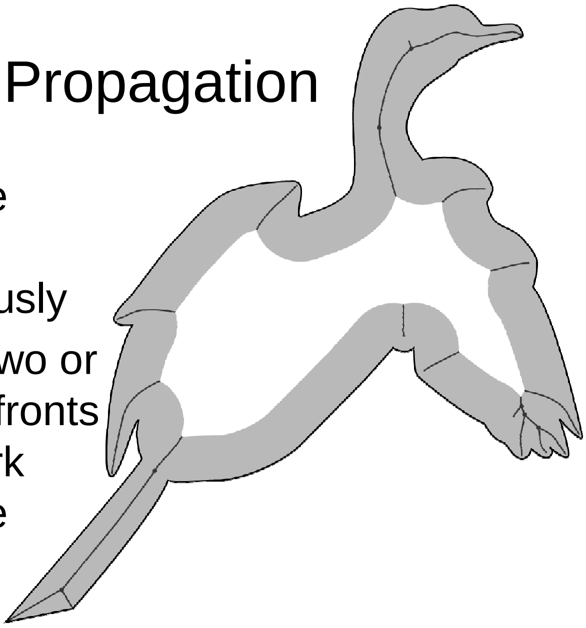

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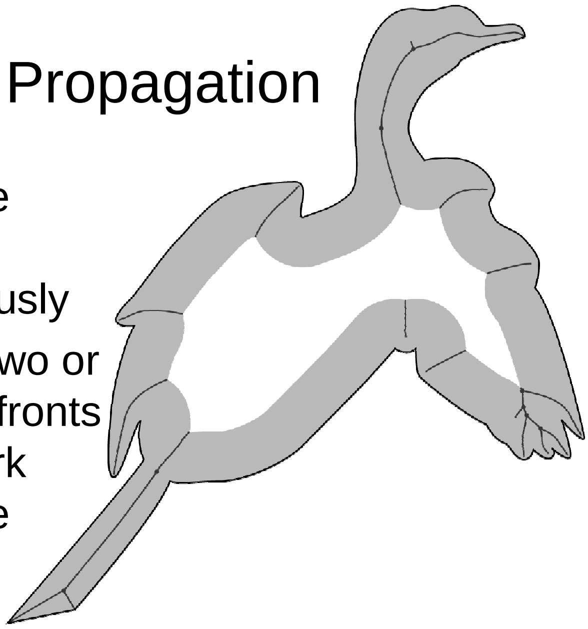

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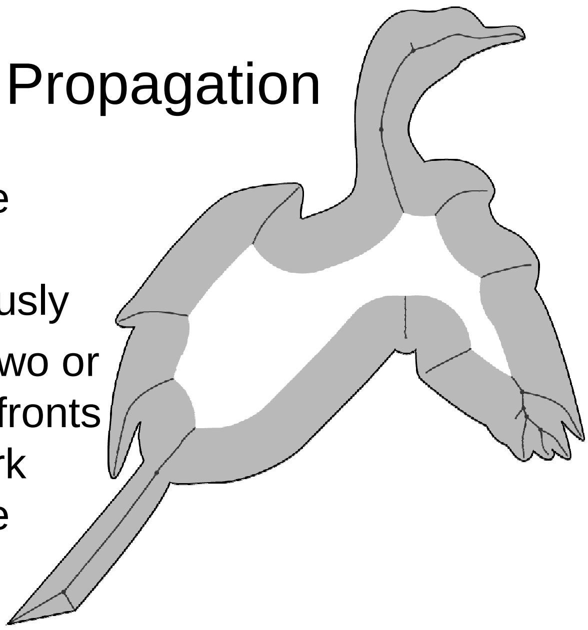

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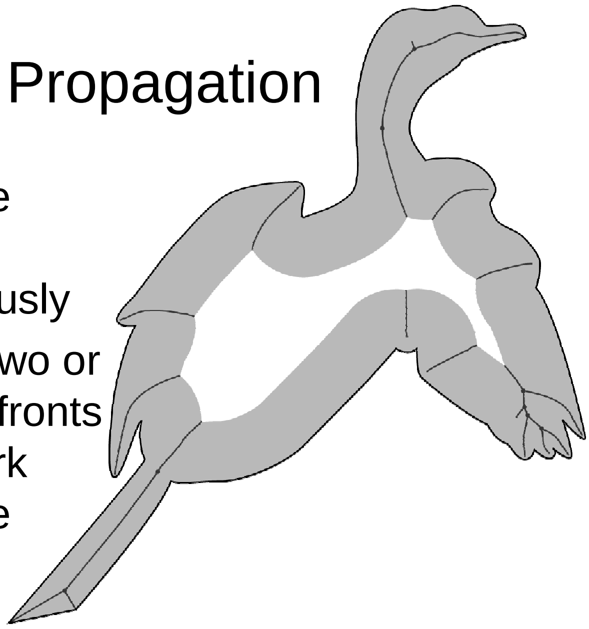

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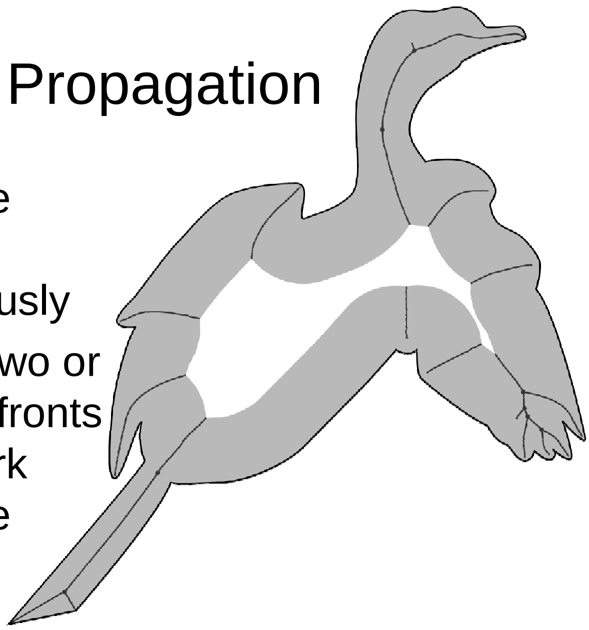

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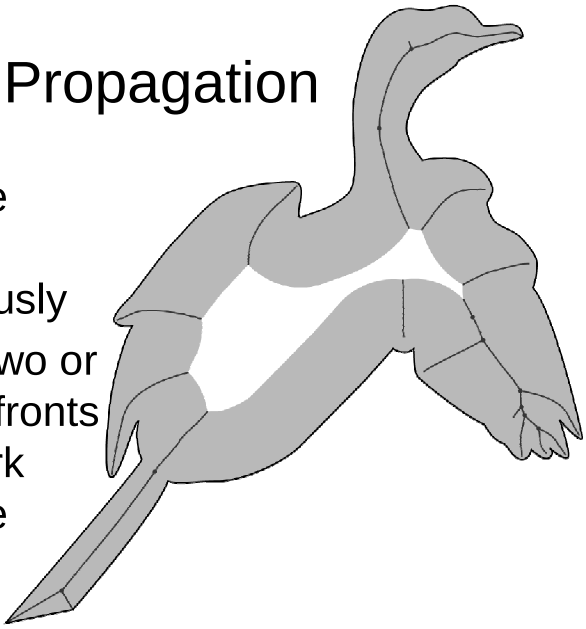

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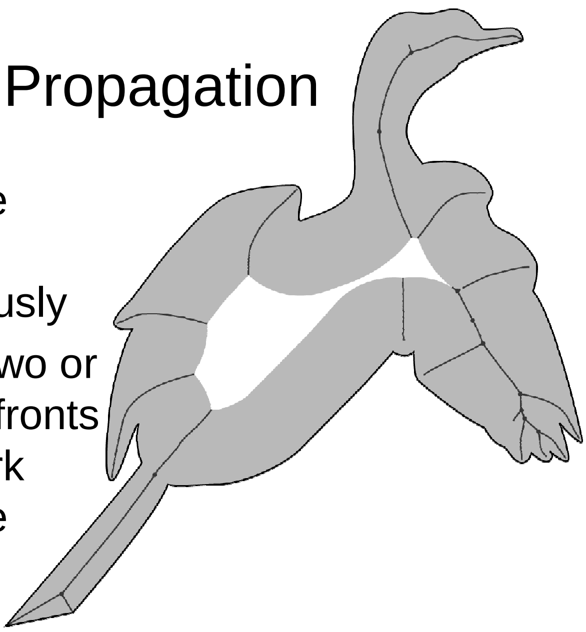

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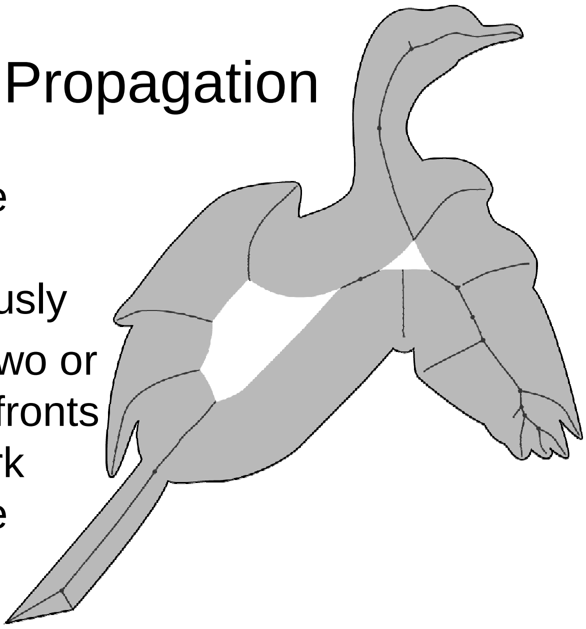

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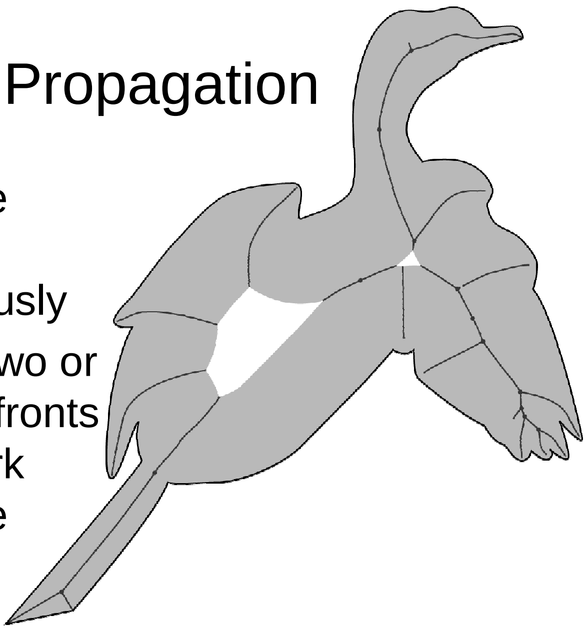

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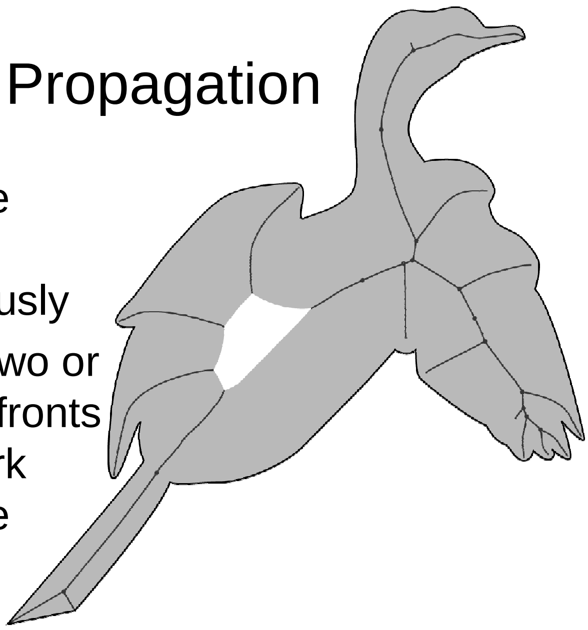

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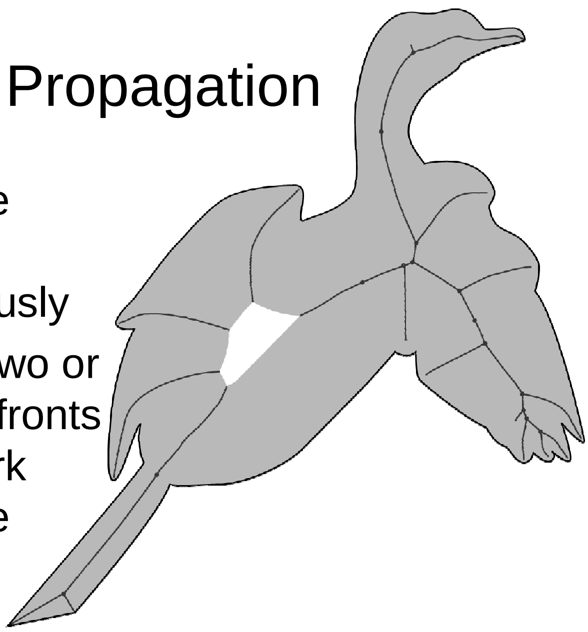

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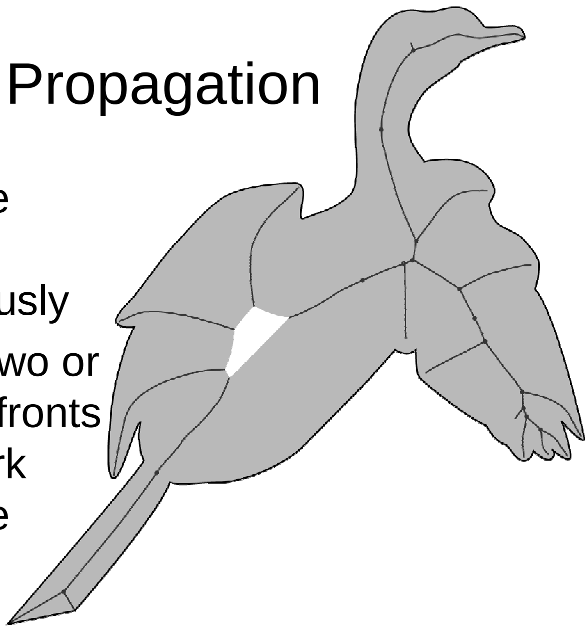

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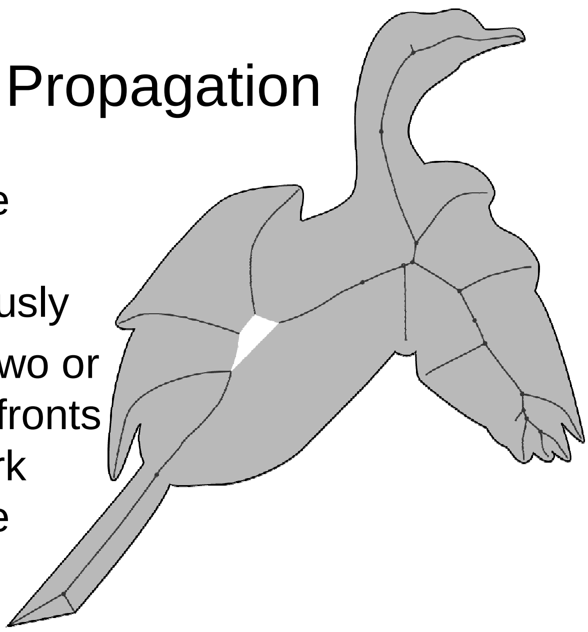

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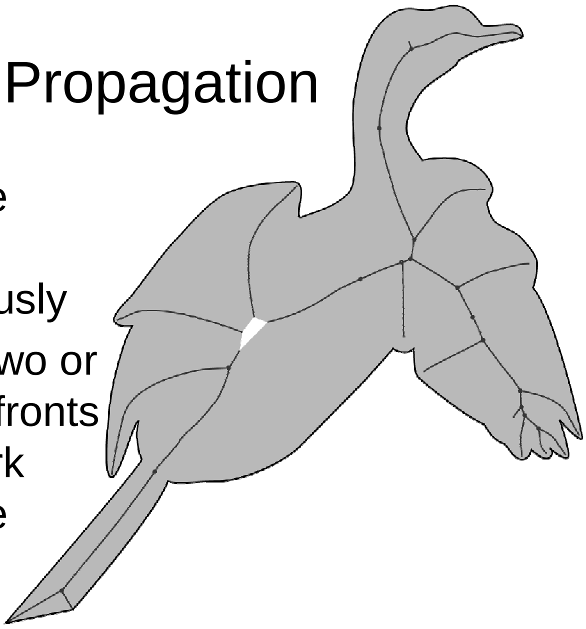

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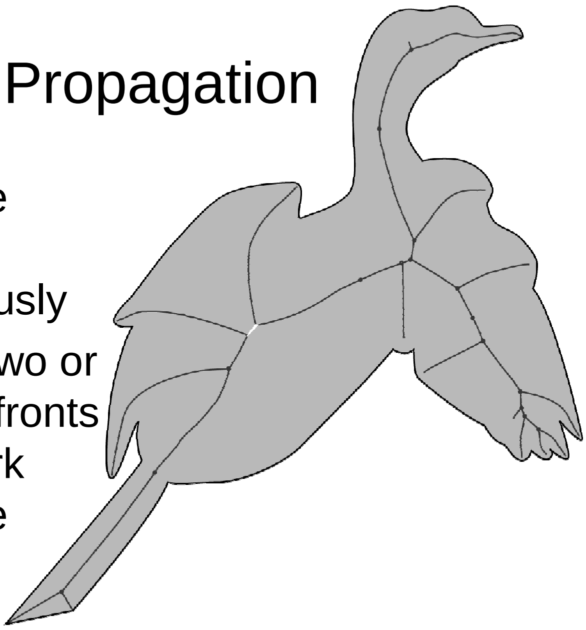

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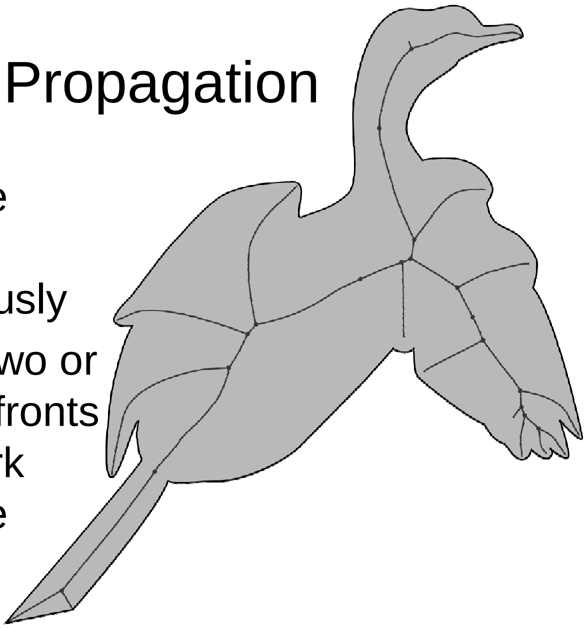
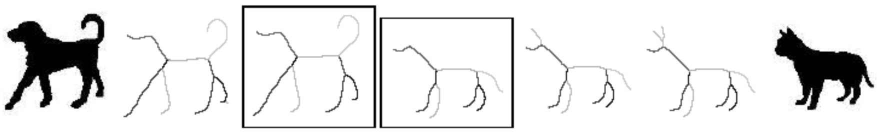
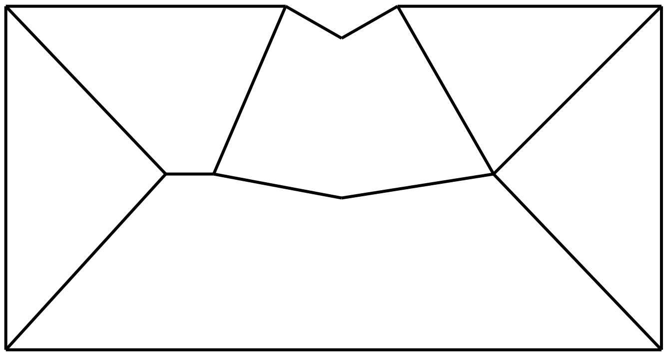




Similarity of structure is more apparent in skeleton than in boundary.

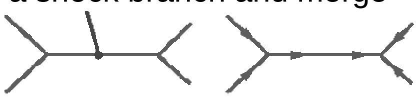
(Sebastian and Kimia)

Sensitive to noise

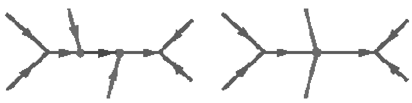


Shock Graph Edit Distance

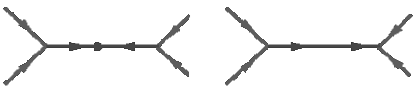
- Splice – Delete a shock branch and merge remaining two



- Contract - deletes a shock branch between degree-three nodes



- Merge - combines two branches at a degree-two node



- Deform - changes the attributes of a shock branch

Skeletons of 3D Objects

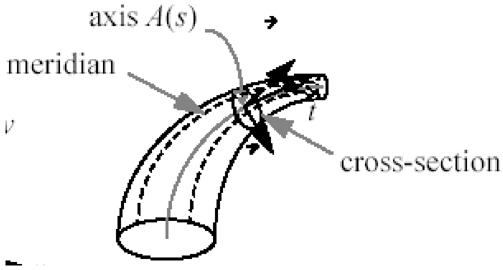
- Grassfire produces a 2D surface.
- Intuitively, skeletons seem 1D.
- Harder to compare 2D surfaces; extract parts, etc….

Generalized Cylinders

Contains

•an axis,

•a cross-section that sweeps along that axis, changing shape.



Problem

- How do you define a 1D skeleton of a 3D shape
- And relate this to the 1D skeleton of 2D image of that shape?

Thin-Plate Splines

A function, *f*, R^2 -> R^2 is a thin-plate spline if:

• **Constraint:** Given corresponding points: X1…Xn and U1…Un, f(Xi)=Ui.

• **Energy:** *f* minimizes the following:

$$\iint_{R^2} \left(\left(\frac{\partial^2 f}{\partial x^2} \right)^2 + \left(\frac{\partial^2 f}{\partial x \partial y} \right)^2 + \left(\frac{\partial^2 f}{\partial y^2} \right)^2 \right) dxdy$$

If we think of this as the amount of bending produced by *f*. Allows arbitrary affine transformation.

It turns out that the solution to this can be written as an affine transformation plus a linear combination of basis elements, where we solve separately for the new x and y values of points.

The basis elements are centered at each of the initial points, Pi, and have the form: U(|Pi-(x,y)|), where U has the form: U(r) = -r^2 log(r).

So, for example, we can take any location, (x,y), and determine its new x coordinate as:

$$x' = f(x,y) = a_1 + a_2 x + a_3 y + \sum_{i=1}^{n} w_i U\left(\left|P_i - (x,y)\right|\right)$$

If we have the constraint that for n points, we know where these points should be before and after the transformation, we get n linear equations, with n+3 unknowns. We can select a solution from these by adding other linear constraints, such as that the sum of the wi's should be 0, and the inner product of the vector of w coefficients and the x and y coordinates of the known points should be 0.

- **Solution:** The function *f* can be computed using straightforward linear algebra. See *Principal Warps: Thin-Plate Splines and the Decomposition of Deformations* by Bookstein, or *Statistical Shape Analysis* by Dryden and Mardia for details.
- **Extension:** Can penalize mismatch of points (using function of || Ui – *f*(Xi)||).
- **Results:** Much like D'Arcy Thompson.