Template Matching – Rigid Motion

- Find transformation to align two images.
- Focus on geometric features
  - (not so much interesting with intensity images)
  - Emphasis on tricks to make this efficient.

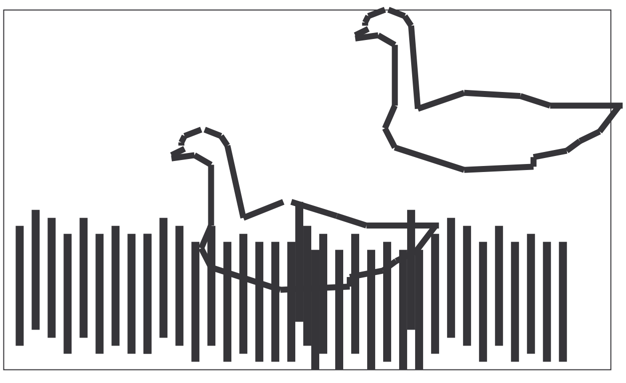
Problem Definition

- An Image is a set of 2D geometric features, along with positions.
- An Object is a set of 2D/3D geometric features, along with positions.
- A pose positions the object relative to the image.
  - 2D Translation; 2D translation + rotation; 2D translation, rotation and scale; planar or 3D object positioned in 3D with perspective or scaled orth.
- The best pose places the object features nearest the image features

Two parts to the problem

- Definition of cost function.
- Search method for finding best pose.
  1. Can phrase this as search among poses.
  2. Or as search among correspondences
  3. There are connections between two.

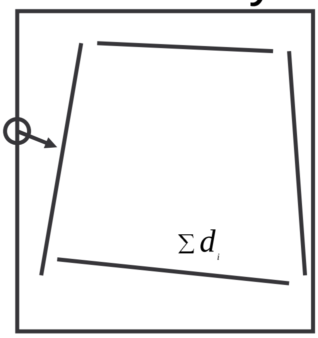
Example



Cost Function

- We look at this first, since it defines the problem.
- Again, no perfect measure;
  - Trade-offs between veracity of measure and computational considerations.
- One-to-one vs. many-to-one
- Bounded error vs. metric

Example: Chamfer Matching Many-to-one, distance



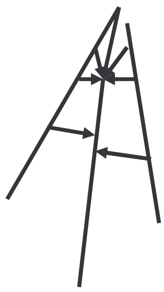
For every edge point in the transformed object, compute the distance to the nearest image edge point. Sum distances.

$$\sum_{i=1}^{n} \min(\| p_i, q_1 \|, \| p_i, q_2 \|, \ldots \| p_i, q_m \|)$$

Main Feature:

• Every model point matches an image point.

• An image point can match 0, 1, or more model points.



Variations

- Sum a different distance
  - $f(d) = d^2$
  - or *Manhattan distance*.
  - $f(d) = 1$ if d < threshold, 0 otherwise.
    - This is called *bounded error*.
- Use maximum distance instead of sum.
  - This is called: *directed Hausdorff distance*.
- Use other features
  - Corners.
  - Lines. Then position and angles of lines must be similar.
    - Model line may be subset of image line.

Other comparisons

- Enforce each image feature can match only one model feature.
- Enforce continuity, ordering along curves.
- These are more complex to optimize.

Pose Search: Standard Optimization Heuristics

- Brute force search with dense sampling.
- Random starting point + gradient descent.
  - Multiple random starting points
  - Stochastic gradient descent
- Any other optimization method you can think of.

Clever Idea 1: Chamfer Matching with the Distance Transform

2	1	0	1	2	3	2
1	0	1	2	3	2	1
0	1	2	3	2	1	0
1	2	3	2	1	0	1
2	3	3	2	1	0	1
3	4	3	2	1	0	1

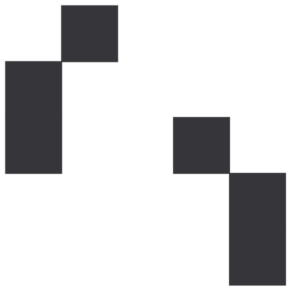
Example: Each pixel has (Manhattan) distance to nearest edge pixel.

D.T. Adds Efficiency

- Compute once.
- Fast algorithms to compute it.
- Makes Chamfer Matching simple.

Then, try all translations of model edges. Add distances under each edge pixel.

That is, correlate edges with Distance Transform



2	1	0	1	2	3	2
1	0	1	2	3	2	1
0	1	2	3	2	1	0
1	2	3	2	1	0	1
2	3	3	2	1	0	1
3	4	3	2	1	0	1

Computing Distance Transform

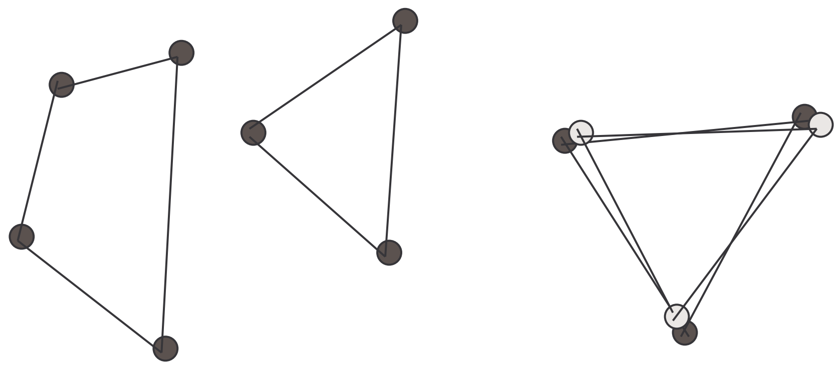
- It's only done once, per problem, not once per pose.
- Basically a shortest path problem.
- Simple solution passing through image once for each distance.
  - First pass mark edges 0.
  - Second, mark 1 anything next to 0, unless it's already marked. Etc….
- Actually, a more clever method requires 2 passes.

Chamfer Matching Complexity

- Brute force approach: for each pose, compare each model point to every image point. O(*pnm*). *p* = number poses, *n* = number of image points, *m* = number of model points.
- With distance transform: compute D.T., then for every pose, sum value under each model edge. O(*s* + *pm*). *s* = number of pixels, which is about same as *p*.

Clever Idea 2: Ransac

- Match enough features in model to features in image to determine pose.
- Examples:
  - match a point and determine translation.
  - match a corner and determine translation and rotation.
  - Points and translation, rotation, scaling?
  - Lines and rotation and translation?



Algorithm 15.4: RANSAC: fitting lines using random sample consensus

Determine:
  n — the smallest number of points required
  k — the number of iterations required
  t — the threshold used to identify a point that fits well
  d — the number of nearby points required
    to assert a model fits well
Until k iterations have occurred
  Draw a sample of n points from the data
    uniformly and at random
  Fit to that set of n points
  For each data point outside the sample
    Test the distance from the point to the line
      against t; if the distance from the point to the line
      is less than t, the point is close
  end
  If there are d or more points close to the line
    then there is a good fit. Refit the line using all
    these points.
end
Use the best fit from this collection, using the
  fitting error as a criterion

(Forsyth & Ponce)

Complexity

- Suppose model has *m* points and image has *n* points. There are *nm* matches.

When we match a model point, there is a 1/n probability this match is right.

If we match *k* model points, probability all are right is approximately $(1/n)^k$.

If we repeat this L times, probability that at least one *pose* is right is:

$$1-\left(1-\left(\frac{1}{n}\right)^{k}\right)^{L}$$

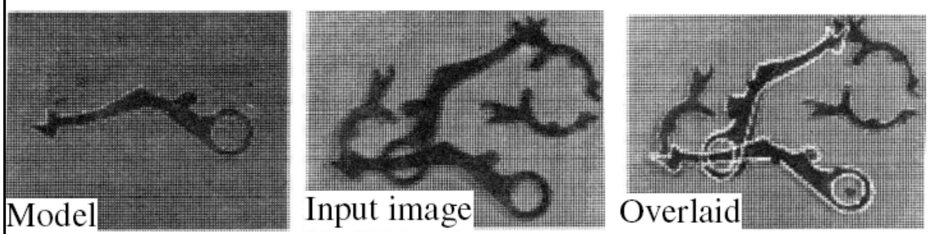


Figure from "Object recognition using alignment," D.P. Huttenlocher and S. Ullman, Proc. Int. Conf. Computer Vision, 1986, copyright IEEE, 1986

The Hough Transform for Lines

- A line is the set of points (x, y) such that:

y = mx + b

- For any (x, y) there is a line in (m,b) space describing the lines through this point. Just let (x,y) be constants and m, b be unknowns.
- Each point gets to vote for each line in the family; if there is a line that has lots of votes, that should be the line passing through the points

Mechanics of the Hough transform

- Construct an array representing m, b
- For each point, render the line y=mx+b into this array, adding one at each cell
- Questions
  - how big should the cells be? (too big, and we cannot distinguish between quite different lines; too small, and noise causes lines to be missed)
- How many lines?
  - count the peaks in the Hough array
- Who belongs to which line?
  - tag the votes
- Can modify voting, peak finding to reflect noise.
- Big problem if noise in Hough space different from noise in image space.

Generalized Hough Transform

Some pros and cons

- Run-time for line finding
  - Complexity of RANSAC n*n*n
  - Complexity of Hough n*d

Error behavior

- Hough handles error with buckets. This gives a larger set of lines consistent with point, but ad-hoc.
- Ransac handles error with threshold. Well-motivated for error in other points, but not for error in first 2 points.
  - But works if we find some 2 points w/ low error.
- Error handling sloppy -> clutter bigger problem.
- Many variations to handle these issues.

Pose: Generalized Hough Transform

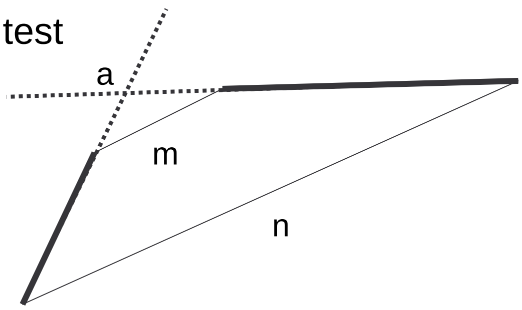
- Like Hough Transform, but for general shapes.
- Example: match one point to one point, and for every rotation of the object its translation is determined.
- Example: match enough features to determine pose, then vote for best pose.

Correspondence: Interpretation Tree Search

- Represent all possible sets of matches as exponential sized tree.
- Each node involves another match
- Wildcard allowed for no matches.
- Prune tree when set of matches incompatible (this seems to imply bounded error).
- Trick: some fast way of evaluating compatability.
- Trick: different tree search algorithms. Best first. A*….

2D Euclidean Transformation

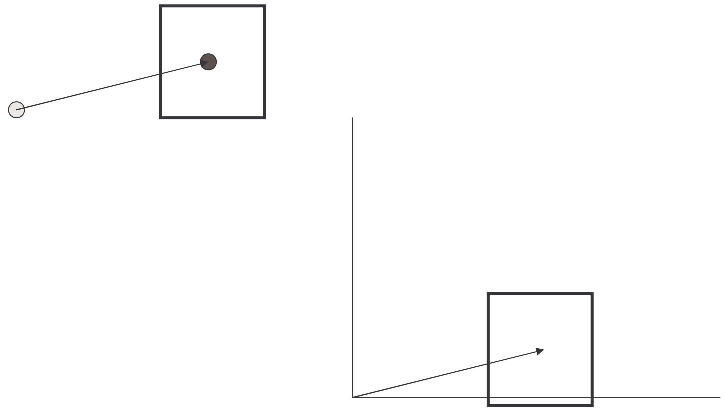
- Check pairwise compatibility
  - Fast
  - Conservative test

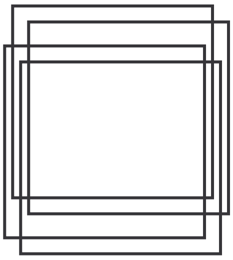


Cass: Correspondence pose duality

- Suppose we match two features with bounded error.
  - There is a set of transformations that fit.
  - For nm matches, nm sets.
  - As these intersect, they carve *transformation space* into regions.
    - Within a region, feasible matches are the same.
    - If sets are convex, #regions is limited.
    - If everything is linear, this becomes easier.

Example: points, 2D translation L-infinity norm





- Every cell is bounded by axial lines.
- Must contain point where two lines intersect.
  - No more than $(nm)^2$ cells.
- If we sample points where all pairs of lines intersect, we sample all cells.

General case

- Can extend to any linear transformation and convex, polygonal error bound.
- Every model point and every error line lead to hyperplane in transformation space.
- These divide transformation space into convex cells. Each has vertices at intersection of d hyperplanes.
- Complexity $(mn)^d$

Can also mix and match

- Alignment, then tree search.
  - continue to add feasible additional matches.
- Greedy heuristics
- RANSAC + distance transform
- …

Summary

- All these methods exponential in dimension of transformation.
- Clever & effective for translation, 2D euclidean.
- Too slow for 3D to 2D recognition
  - Grouping heuristics
    - Roberts – group quadrilaterals, then alignment.
    - Lowe – Perceptually salient grouping based on parallelism, proximity, ….