Which Operations are P Closed Under? Which Operations are NP Closed Under?

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We will look look at what is known about closure of P and of NP under the following operations:

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Union

We will look look at what is known about closure of P and of NP under the following operations:

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- Union
- Intersection

We will look look at what is known about closure of P and of NP under the following operations:

- Union
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- Union
- Intersection
- Complement
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- Union
- Intersection
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- Kleene star

# **Closure Properties of P**

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**Thm** If  $L_1 \in P$  and  $L_2 \in P$  then  $L_1 \cup L_2 \in P$ .

**Thm** If  $L_1 \in P$  and  $L_2 \in P$  then  $L_1 \cup L_2 \in P$ .  $L_1 \in P$  via TM  $M_1$  which works in time  $p_1(n)$ .

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- 1. Input(x) (We assume |x| = n.) Let  $x = x_1 \cdots x_n$
- 2. For  $0 \le i \le n$ 
  - 2.1 Run  $M_1(x_1 \cdots x_i)$  and  $M_2(x_{i+1} \cdots x_n)$ . If both say Y then output Y and STOP. (Time:  $p_1(i) + p_2(n-i) \le p_1(n) + p_2(n)$ .)

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### **Closure P Under Concatenation**

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This algorithm takes  $\leq (n + 1) \times (p_1(n) + p_2(n))$  which is poly. **Note** Key is that the set of polynomials is closed under addition and mult by *n*.

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- 2. Run M(x). Answer is b.

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**Note** No note needed.

 $L \in \mathrm{P} \to L^* \in \mathrm{P}$  ?

#### **Attempt Proof**

First lets talk about what you should not do.

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 Break string into n piece: <sup>n</sup><sub>n</sub> ways to do this.

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What is another name for this?

- **B** is Bill, **D** is Darling.
- **B:** D, how many subsets are there of  $\{1, \ldots, n\}$ ?

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**B:** Another Way: 1 is IN or OUT, 2 is IN or OUT, etc, so  $2^n$ .

- **B** is Bill, **D** is Darling.
- **B:** D, how many subsets are there of  $\{1, \ldots, n\}$ ?
- D: You can either choose 0 elements or choose 1 element, so

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- D: Really!
- B: Yes, really!

Vote

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#### Vote

1. P is closed under \*. Someone has a trick or hard math or a computer program to help do this. Fire and Brimstone speech about lower bounds to follow.

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Answer on Next Slide

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**Original Problem** Given  $x = x_1 \cdots x_n$  want to know if  $x \in L^*$  **New Problem** Given  $x = x_1 \cdots x_n$  want to know:  $e \in L^*$   $x_1 \in L^*$   $x_1x_2 \in L^*$   $\vdots$  $x_1x_2 \cdots x_n \in L^*$ .

#### P is Closed Under \*

The technique of looking at **all** ways to break up x into pieces takes roughly  $2^n$  steps, so we need to do something clever. **Dynamic Programming** We solve a harder problem but get lots of information we don't need in the process. **Original Problem** Given  $x = x_1 \cdots x_n$  want to know if  $x \in L^*$ **New Problem** Given  $x = x_1 \cdots x_n$  want to know:  $e \in L^*$  $x_1 \in L^*$  $x_1x_2 \in L^*$  $x_1x_2\cdots x_n \in L^*$ . **Intuition**  $x_1 \cdots x_i \in L^*$  IFF it can be broken into TWO pieces, the first one in  $L^*$ , and the second in L.

A[i] stores if  $x_1 \cdots x_i$  is in  $L^*$ . *M* is poly-time Alg for *L*, poly *p*.

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# What Operations is NP Closed Under?

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**Thm** If  $L_1 \in NP$  and  $L_2 \in NP$  then  $L_1 \cup L_2 \in NP$ .



**Thm** If  $L_1 \in NP$  and  $L_2 \in NP$  then  $L_1 \cup L_2 \in NP$ .  $L_1 = \{x : (\exists y_1)[|y_1| = p_1(|x|) \land (x, y_1) \in B_1]\}$ 

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The following defines  $L_1 \cup L_2$  in an NP-way.  
 $L_1 \cup L_2 = \{x : (\exists y) \in L_2 \in A : (\exists y) \in A \}$ 

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Witness  $|y| = p_1(|x|) + p_2(|x|) + 1$  is short. Verification  $(x, y_1) \in B_1 \lor (x, y_2) \in B_2$ , is quick.

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#### **Closure of NP Under Intersection**

#### **Thm** If $L_1 \in NP$ and $L_2 \in NP$ then $L_1 \cap L_2 \in NP$ .

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# **Closure of NP Under Intersection**

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1}

 $x = x_1 x_2$  $|y_1| = p_1(|x_1|)$  $|y_2| = p_2(|x_2|)$  $(x_1, y_1) \in B_1$  $(x_2, y_2) \in B_2$ 

#### **Closure of NP Under \***

**Thm** If  $L \in NP$  then  $L^* \in NP$ .



# **Closure of NP Under \***

Thm If  $L \in NP$  then  $L^* \in NP$ .  $L = \{x : (\exists y)[|y| = p(|x|) \land (x, y) \in B]\}$ 



**Thm** If  $L \in NP$  then  $L^* \in NP$ .  $L = \{x : (\exists y)[|y| = p(|x|) \land (x, y) \in B]\}$ The following defines  $L^*$  in an NP-way

$$\{x: (\exists z_1,\ldots,z_k,y_1,\ldots,y_k)\}$$

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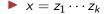
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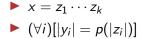
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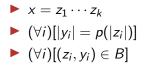
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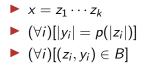
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Vote

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#### Vote

1. There is a proof that if  $L \in NP$  then  $\overline{L} \in NP$ . (Hence NP is closed under complementation and we know this.)

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#### Vote

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Answer on next slide.

Most Complexity Theorists think  $\operatorname{NP}$  is  $\operatorname{\textbf{not}}$  closed under complementation.

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**Contrast** Alice is all powerful, Bob is Poly Time.

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It is thought that there is no way for Alice to do this.