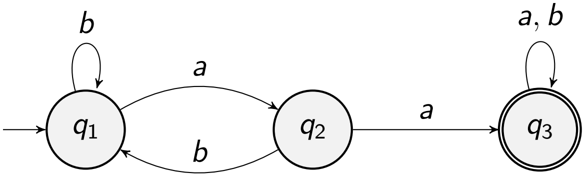
Deterministic Finite Automata (DFA): Closure Properties

Two Fine Languages

The language L_a is the set of words over $\{a, b\}$ with two consecutive a's. DFA for L_a:



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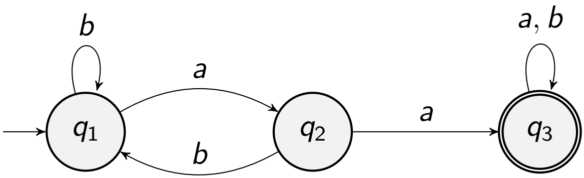
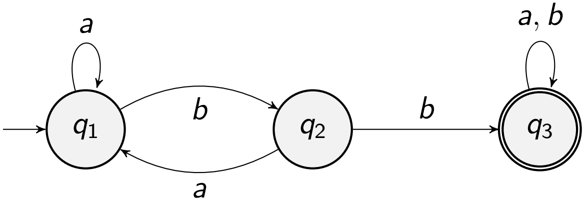


The language L_b is the set of words over $\{a, b\}$ with two consecutive b's. DFA for L_b:



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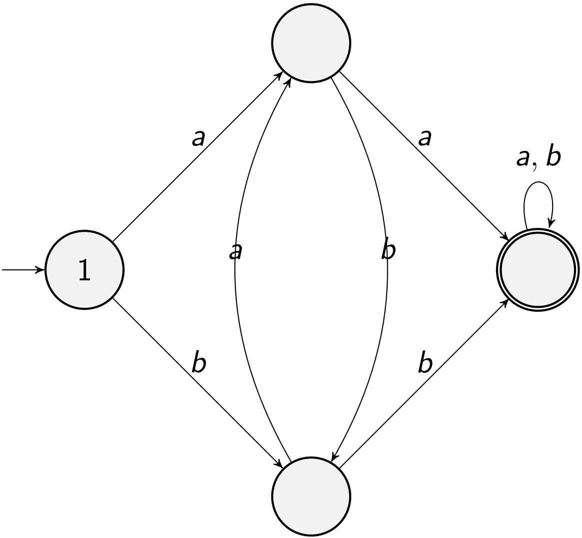
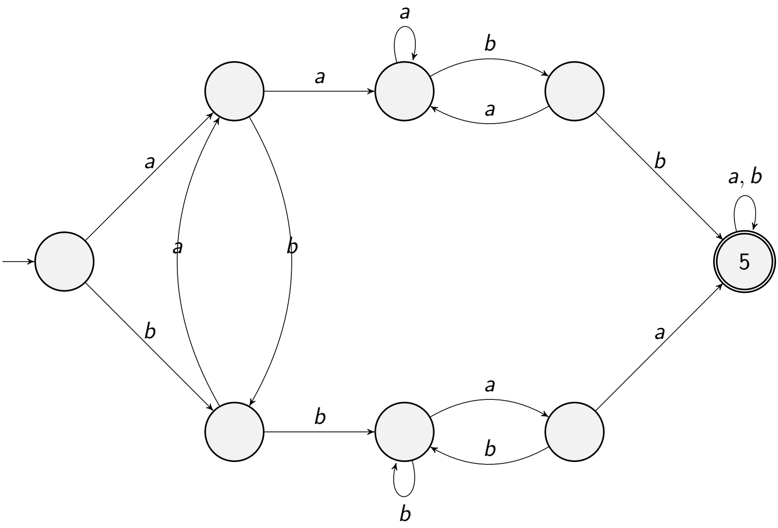


Intersection: $L_a \cap L_b$



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Can we do this automatically?

Idea: First check two a's then check two b's. **No!**

Must do two checks in parallel by "running both machines at once".

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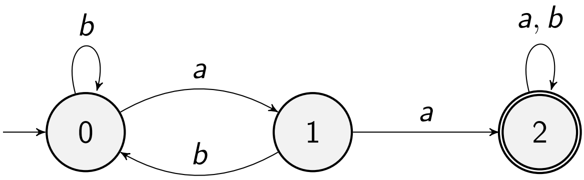

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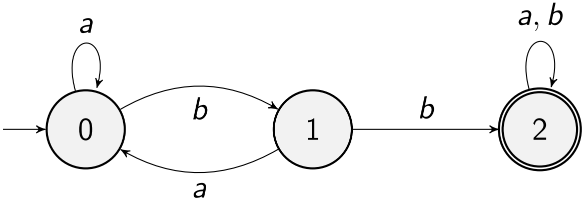
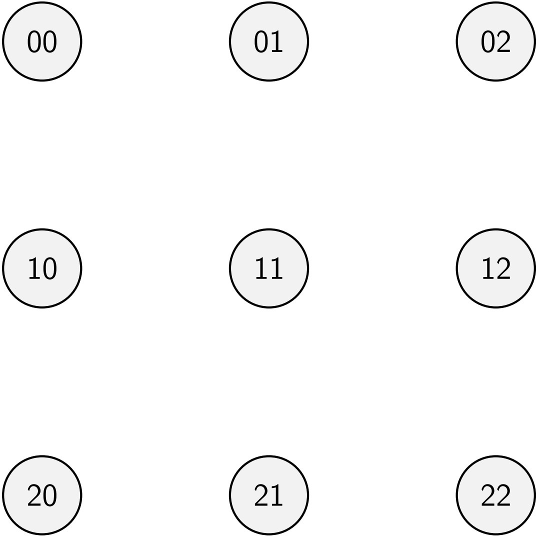


Grid



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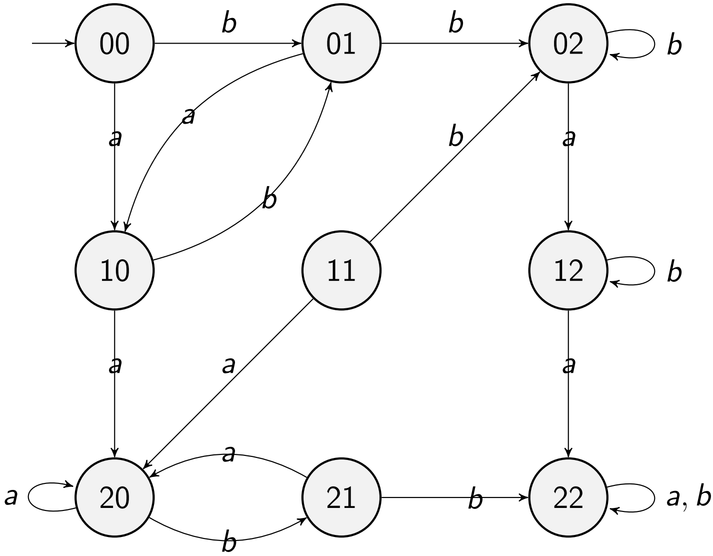

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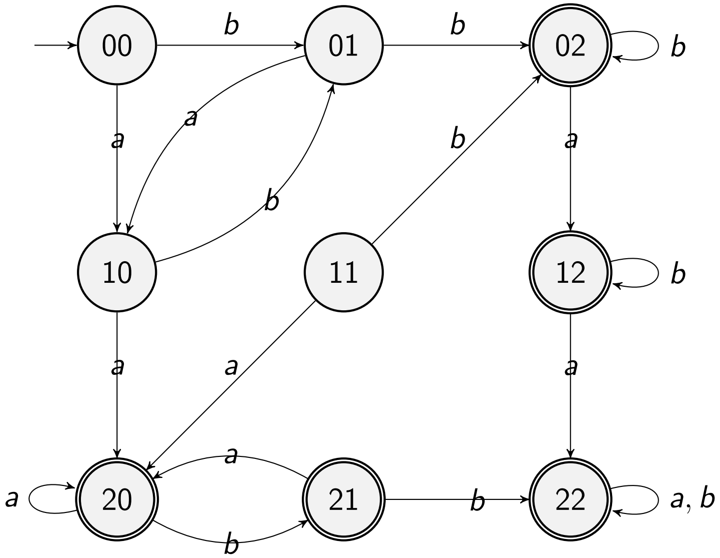

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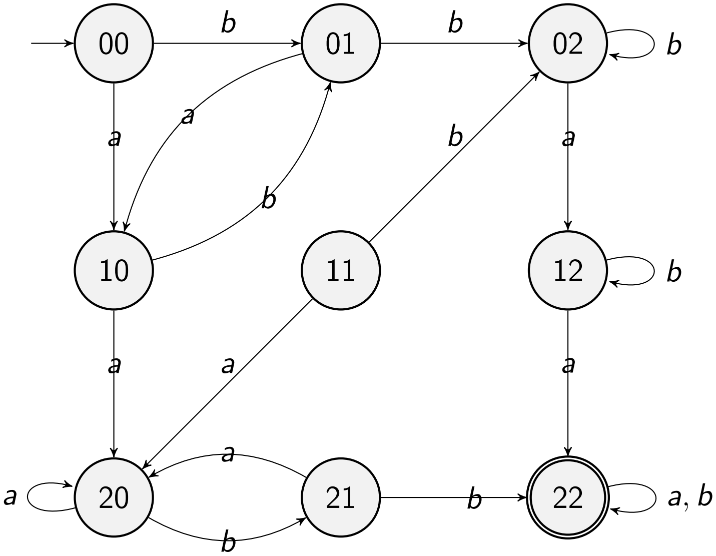

Regular Lang Closed Under Union

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$$(Q_1 \times Q_2, \Sigma, \delta, (s_1, s_2), F)$$

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Note The number of states in DFA for $L_1 \cup L_2$ is $n_1 n_2$.

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How do you compliment a regular language?

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Complement The complement of L is $\Sigma^* - L$.

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Informally Swap the final and non-final states.

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Note If DFA for L has n states then DFA for $\overline{L}$ has n states.

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IF L_1, L_2 are regular then $L_1 \cdot L_2$ is regular.

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Summary of Closure Properties and Proofs

X means **Can't Prove Easily**

$n_1 + n_2$ (and similar) is number of states in new machine if L_i reg via n_i-state machine.

Closure Property	DFA
$L_1 \cup L_2$	$n_1 n_2$
$L_1 \cap L_2$	$n_1 n_2$
$L_1 \cdot L_2$	X
$\overline{L}$	n
L^*	X

BILL, STOP RECORDING LECTURE!!!!

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