

BILL, RECORD LECTURE!!!!

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**A Lang that has a TINY
CFG a MEDIUM NFA
and a LARGE DFA**

DFA and NFA and CFG

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3. There is an NFA for L_n with $n + 2$ states.

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2. Every Chomsky Normal Form CFG for L_n has $\geq \lg n$ size. We will not prove this.

Upper and Lower Bounds for DFA for L_n

Upper and Lower Bounds for NFA for L_n

Upper and Lower Bounds for CFG for L_n

CFG for Σ^*

Lemma There is a CFG for Σ^* of size $O(1)$.

$$S \rightarrow AS$$

$$S \rightarrow BS$$

$$S \rightarrow e$$

$$A \rightarrow a$$

$$B \rightarrow b$$

CFG for Σ^n of Size $\leq 2 \lg n$

We first do this when $n = 2^m$, a power of 2.

$$S \rightarrow A_1 A_1$$

$$A_1 \rightarrow A_2 A_2$$

$$A_2 \rightarrow A_3 A_3$$

\vdots

$$A_{m-1} \rightarrow A_m A_m$$

$$A_m \rightarrow a \mid b$$

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Can show that

$$S \Rightarrow A_1 A_1 = A_1^2 = A_1^{2^1}$$

$$S \Rightarrow A_2 A_2 A_2 A_2 = A_2^4 = A_2^{2^2}$$

$$(\forall 1 \leq i \leq m) [S \Rightarrow A_i^{2^i}]$$

$$\text{In particular } S \Rightarrow A_m^{2^m} = A_m^n$$

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Since A_m is the only nonterminal that can generate terminals, and A_m^n is the only string of A_m 's that can be produced, and

$A_m \rightarrow a \mid b$, we have

$L(G) = \Sigma^n$.

CFG for Σ^n

What if n is not a power of 2?

Leave it to you to show there is a Chomsky Normal Form Grammar of Size $\leq 2 \lg n$.

CFG for $\Sigma^* a \Sigma^n$

Let G_1 be grammar for Σ^* . Let S_1 be its start symbol. Size 5

Let G_2 be grammar for Σ^n . Let S_2 be its start symbol. Size $\leq 2 \lg n$.

Relabel if needed so they use disjoint set of nonterminals.

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$S \rightarrow S_1 X$

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Size $\leq 3 + 5 + 2 \lg n = 8 + 2 \lg n$.