Deterministic Finite Automata (DFA)

DFAs

DFAs

Three Examples

Standard Conventions

Standard Conventions

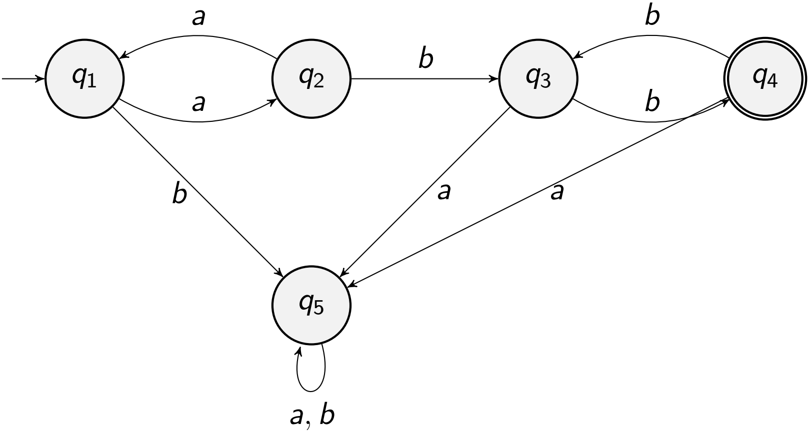
1. The state that has an arrow pointing to it (from nowhere, not from another state) is the **start** state.

Standard Conventions

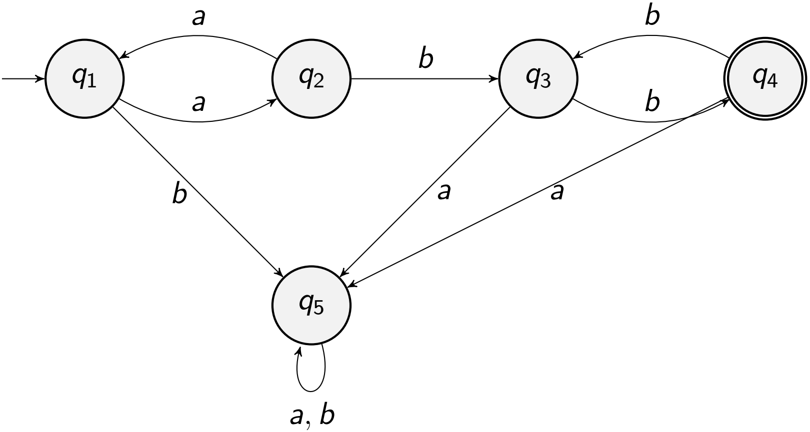
1. The state that has an arrow pointing to it (from nowhere, not from another state) is the **start** state.
2. The states that are circled are **final states**. If the machine ends up there, then the string is accepted.

DFA Diagram: A First Example

DFA Diagram: A First Example

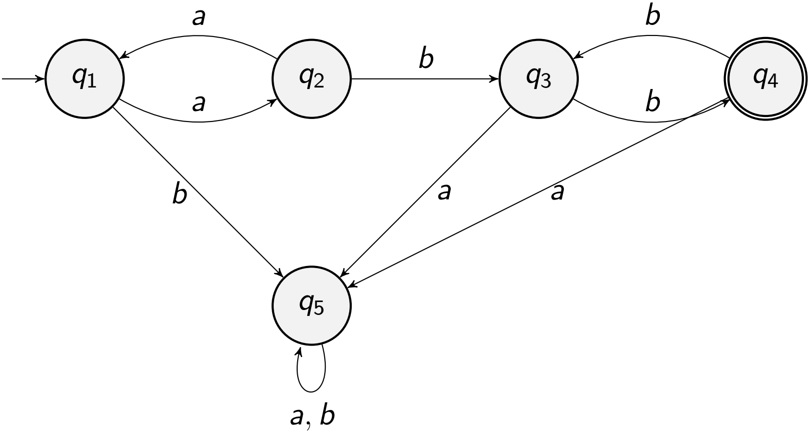


DFA Diagram: A First Example



What is the language?

DFA Diagram: A First Example

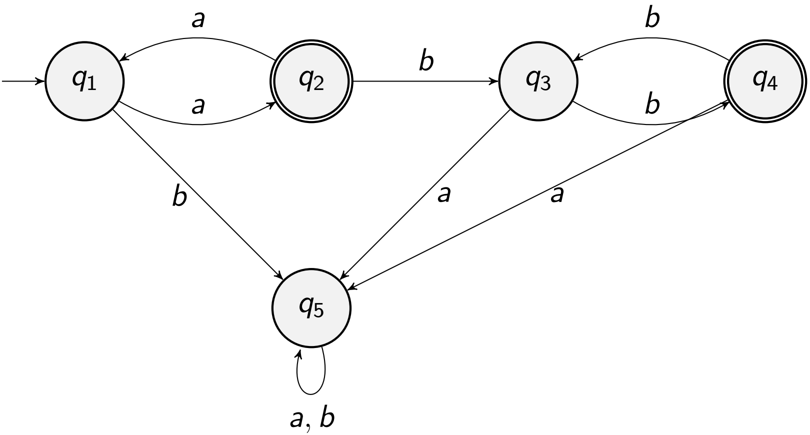


What is the language?

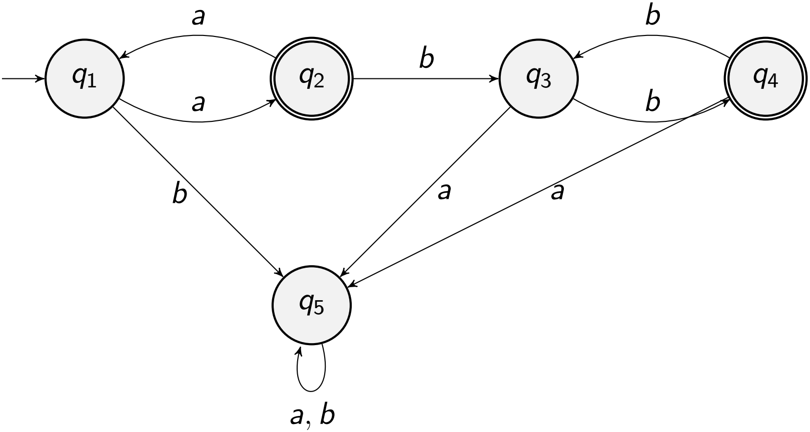
Odd number of *a*'s followed by an even number of *b*'s, but at least two.

DFA Diagram: A Second Example

DFA Diagram: A Second Example

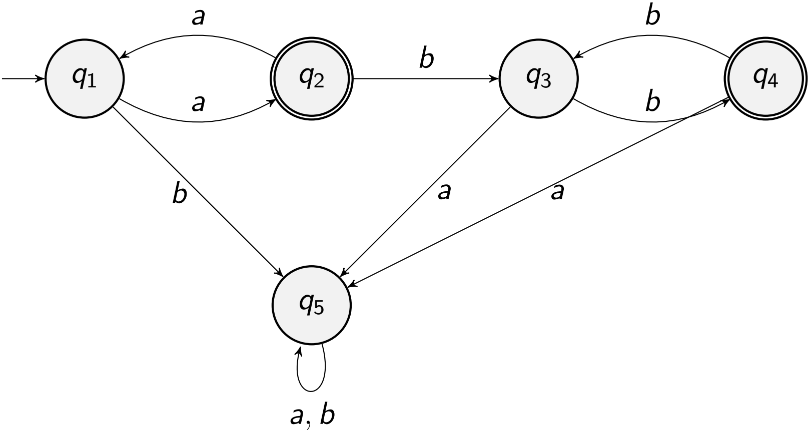


DFA Diagram: A Second Example



What is the language?

DFA Diagram: A Second Example

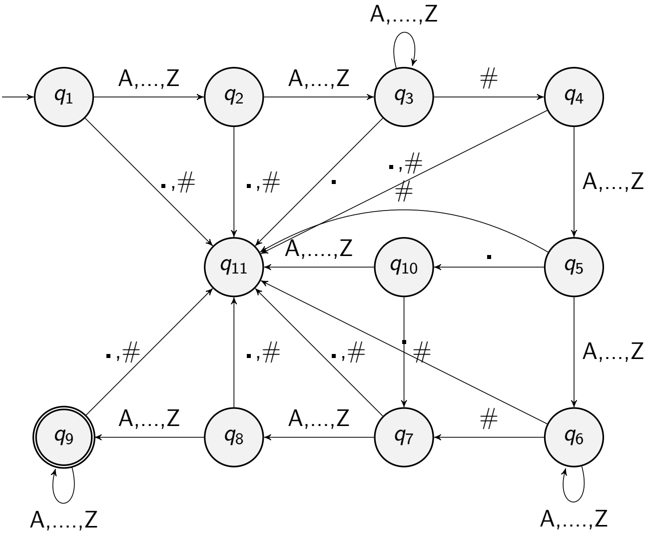


What is the language?

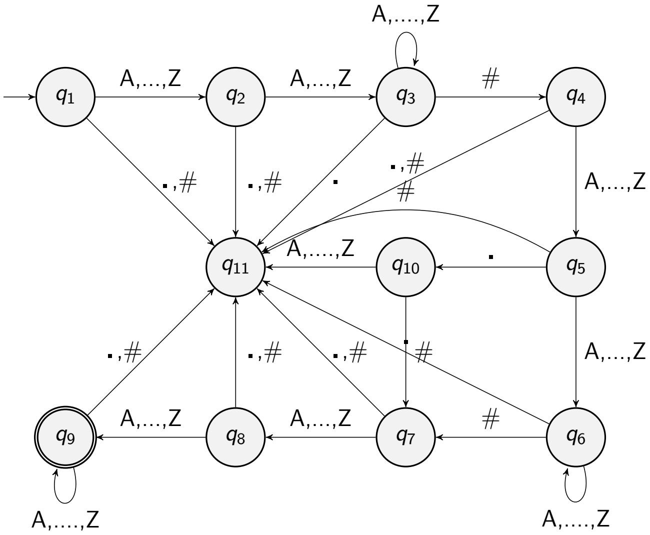
Odd number of *a*'s followed by an even number of *b*'s.

DFA Diagram: A Third Example

DFA Diagram: A Third Example

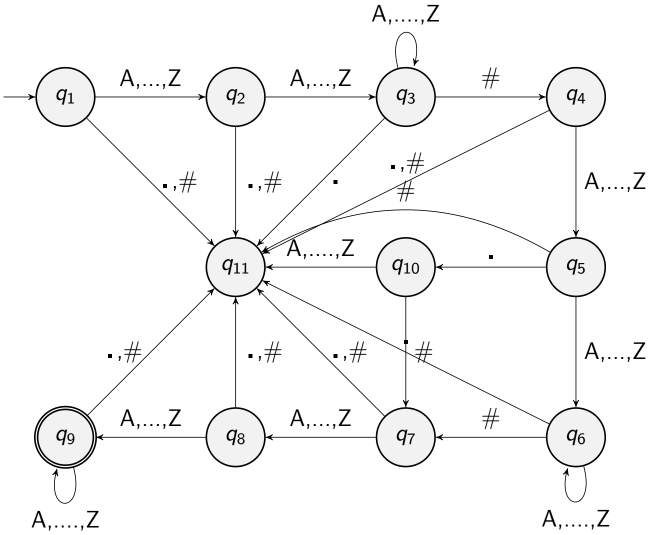


DFA Diagram: A Third Example



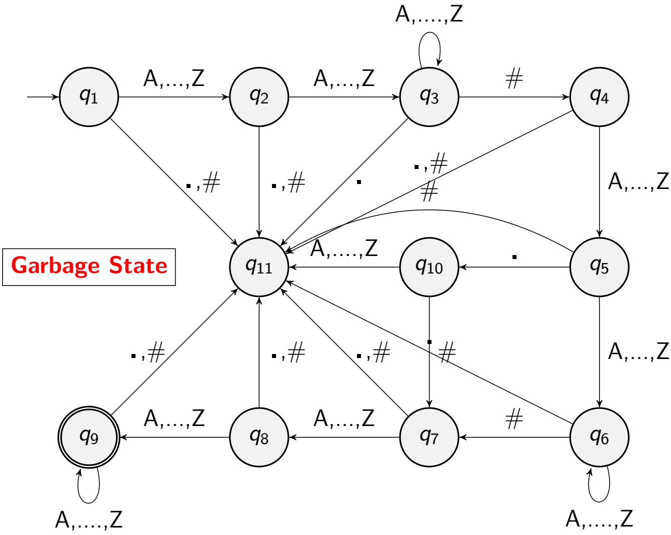
What is the language?

DFA Diagram: A Third Example



What is the language? **Messy**

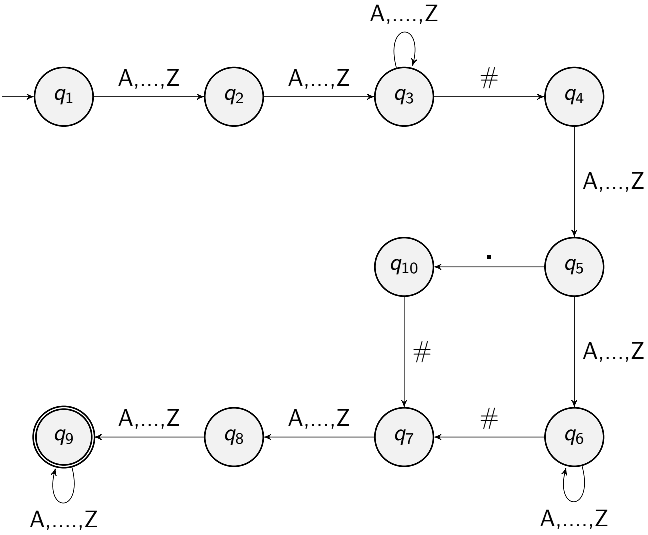
DFA Diagram: A Third Example



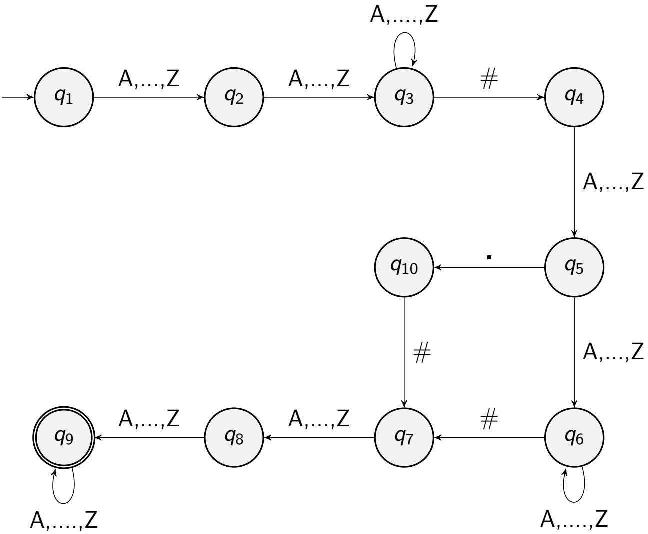
What is the language? Messy

Third Example without Garbage State

Third Example without Garbage State



Third Example without Garbage State



What is the language?

Short Detour

Short Detour

Modular Arithmetic

Modular Arithmetic: Definitions

Modular Arithmetic: Definitions

- $x \equiv y \pmod{N}$ if and only if N divides $x - y$.

Modular Arithmetic: Definitions

- $x \equiv y \pmod{N}$ if and only if N divides $x - y$.
- $25 \equiv 35 \pmod{10}$.

Modular Arithmetic: Definitions

- $x \equiv y \pmod{N}$ if and only if N divides $x - y$.
- $25 \equiv 35 \pmod{10}$.
- $100 \equiv 2 \pmod{7}$ since $100 = 7 \times 14 + 2$.

Modular Arithmetic II: Convention

Common usage:

$$100 \equiv 2 \pmod 7$$

Modular Arithmetic II: Convention

Common usage:

$$100 \equiv 2 \pmod{7}$$

Commonly if we are in mod n we have a large number on the left and then a number between 0 and $n-1$ on the right.

Modular Arithmetic II: Convention

Common usage:

$$100 \equiv 2 \pmod{7}$$

Commonly if we are in mod n we have a large number on the left and then a number between 0 and $n-1$ on the right.

When dealing with mod n we assume the entire universe is $\{0, 1, \ldots, n-1\}$.

Modular Arithmetic: $+, -, \times$

$\equiv$ is mod 26 for this slide. (This slide is from CMSC456.)

Modular Arithmetic: $+, -, \times$

$\equiv$ is mod 26 for this slide. (This slide is from CMSC456.)

1. Addition: $x + y$ is easy: wrap around. E.g., $20 + 10 \equiv 30 \equiv 4$. Only use the number 30 as an intermediary value on the way to the real answer.

Modular Arithmetic: $+, -, \times$

$\equiv$ is mod 26 for this slide. (This slide is from CMSC456.)

1. Addition: $x + y$ is easy: wrap around. E.g., $20 + 10 \equiv 30 \equiv 4$. Only use the number 30 as an intermediary value on the way to the real answer.
2. $-7 \equiv x$ where $0 \leq x \leq 25$.

Modular Arithmetic: $+, -, \times$

$\equiv$ is mod 26 for this slide. (This slide is from CMSC456.)

1. Addition: $x + y$ is easy: wrap around. E.g., $20 + 10 \equiv 30 \equiv 4$. Only use the number 30 as an intermediary value on the way to the real answer.
2. $-7 \equiv x$ where $0 \leq x \leq 25$.
   Pedantic: $-y$ is the number such that $y + (-y) \equiv 0$.

Modular Arithmetic: $+, -, \times$

$\equiv$ is mod 26 for this slide. (This slide is from CMSC456.)

1. Addition: $x + y$ is easy: wrap around. E.g., $20 + 10 \equiv 30 \equiv 4$. Only use the number 30 as an intermediary value on the way to the real answer.
2. $-7 \equiv x$ where $0 \le x \le 25$.
   Pedantic: $-y$ is the number such that $y + (-y) \equiv 0$.
   $-7 \equiv 19 \pmod{26}$ because $19 + 7 \equiv 0 \pmod{26}$.

Modular Arithmetic: $+, -, \times$

$\equiv$ is mod 26 for this slide. (This slide is from CMSC456.)

1. Addition: $x + y$ is easy: wrap around. E.g., $20 + 10 \equiv 30 \equiv 4$. Only use the number 30 as an intermediary value on the way to the real answer.
2. $-7 \equiv x$ where $0 \le x \le 25$.
   Pedantic: $-y$ is the number such that $y + (-y) \equiv 0$.
   $-7 \equiv 19 \pmod{26}$ because $19 + 7 \equiv 0 \pmod{26}$.
   Shortcut: $-y \equiv 26 - y$.

Modular Arithmetic: $+, -, \times$

$\equiv$ is mod 26 for this slide. (This slide is from CMSC456.)

1. Addition: $x + y$ is easy: wrap around. E.g., $20 + 10 \equiv 30 \equiv 4$. Only use the number 30 as an intermediary value on the way to the real answer.
2. $-7 \equiv x$ where $0 \leq x \leq 25$.
   Pedantic: $-y$ is the number such that $y + (-y) \equiv 0$.
   $-7 \equiv 19 \pmod{26}$ because $19 + 7 \equiv 0 \pmod{26}$.
   Shortcut: $-y \equiv 26 - y$.
3. Mult: xy is easy: wrap around. E.g., $20 \times 10 \equiv 200 \equiv 18$.

Modular Arithmetic: $+, -, \times$

$\equiv$ is mod 26 for this slide. (This slide is from CMSC456.)

1. Addition: $x + y$ is easy: wrap around. E.g., $20 + 10 \equiv 30 \equiv 4$. Only use the number 30 as an intermediary value on the way to the real answer.
2. $-7 \equiv x$ where $0 \le x \le 25$.
   Pedantic: $-y$ is the number such that $y + (-y) \equiv 0$.
   $-7 \equiv 19 \pmod{26}$ because $19 + 7 \equiv 0 \pmod{26}$.
   Shortcut: $-y \equiv 26 - y$.
3. Mult: xy is easy: wrap around. E.g., $20 \times 10 \equiv 200 \equiv 18$.
   Shortcut to avoid big numbers:

Modular Arithmetic: $+, -, \times$

$\equiv$ is mod 26 for this slide. (This slide is from CMSC456.)

1. Addition: $x + y$ is easy: wrap around. E.g., $20 + 10 \equiv 30 \equiv 4$. Only use the number 30 as an intermediary value on the way to the real answer.
2. $-7 \equiv x$ where $0 \le x \le 25$.
   Pedantic: $-y$ is the number such that $y + (-y) \equiv 0$.
   $-7 \equiv 19 \pmod{26}$ because $19 + 7 \equiv 0 \pmod{26}$.
   Shortcut: $-y \equiv 26 - y$.
3. Mult: xy is easy: wrap around. E.g., $20 \times 10 \equiv 200 \equiv 18$.
   Shortcut to avoid big numbers:

$$20 \times 10 \equiv -6 \times 10 \equiv -2 \times 30 \equiv -2 \times 4 \equiv -8 \equiv 18.$$

Modular Arithmetic: $+, -, \times$

$\equiv$ is mod 26 for this slide. (This slide is from CMSC456.)

1. Addition: $x + y$ is easy: wrap around. E.g., $20 + 10 \equiv 30 \equiv 4$. Only use the number 30 as an intermediary value on the way to the real answer.
2. $-7 \equiv x$ where $0 \le x \le 25$.
   Pedantic: $-y$ is the number such that $y + (-y) \equiv 0$.
   $-7 \equiv 19 \pmod{26}$ because $19 + 7 \equiv 0 \pmod{26}$.
   Shortcut: $-y \equiv 26 - y$.
3. Mult: xy is easy: wrap around. E.g., $20 \times 10 \equiv 200 \equiv 18$.
   Shortcut to avoid big numbers:

$$20 \times 10 \equiv -6 \times 10 \equiv -2 \times 30 \equiv -2 \times 4 \equiv -8 \equiv 18.$$

4. Division: Next Slide.

Modular Arithmetic: $\div$

$\equiv$ is mod 26 for this slide.
$\frac{1}{3} \equiv x$ where $0 \le x \le 25$.

Modular Arithmetic: $\div$

$\equiv$ is mod 26 for this slide.
$\frac{1}{3} \equiv x$ where $0 \leq x \leq 25$.
Pedantic: $\frac{1}{y}$ is the number such that $y \times \frac{1}{y} \equiv 1$.

Modular Arithmetic: $\div$

$\equiv$ is mod 26 for this slide.
$\frac{1}{3} \equiv x$ where $0 \leq x \leq 25$.
Pedantic: $\frac{1}{y}$ is the number such that $y \times \frac{1}{y} \equiv 1$.
$\frac{1}{3} \equiv 9$ since $9 \times 3 = 27 \equiv 1$.

Modular Arithmetic: $\div$

$\equiv$ is mod 26 for this slide.
$\frac{1}{3} \equiv x$ where $0 \leq x \leq 25$.
Pedantic: $\frac{1}{y}$ is the number such that $y \times \frac{1}{y} \equiv 1$.
$\frac{1}{3} \equiv 9$ since $9 \times 3 = 27 \equiv 1$.
Shortcut:

Modular Arithmetic: $\div$

$\equiv$ is mod 26 for this slide.
$\frac{1}{3} \equiv x$ where $0 \leq x \leq 25$.
Pedantic: $\frac{1}{y}$ is the number such that $y \times \frac{1}{y} \equiv 1$.
$\frac{1}{3} \equiv 9$ since $9 \times 3 = 27 \equiv 1$.
Shortcut: there is an algorithm that finds $\frac{1}{y}$ quickly.

Modular Arithmetic: $\div$

$\equiv$ is mod 26 for this slide.
$\frac{1}{3} \equiv x$ where $0 \leq x \leq 25$.
Pedantic: $\frac{1}{y}$ is the number such that $y \times \frac{1}{y} \equiv 1$.
$\frac{1}{3} \equiv 9$ since $9 \times 3 = 27 \equiv 1$.
Shortcut: there is an algorithm that finds $\frac{1}{y}$ quickly.
We will NOT study the algorithm later.

Modular Arithmetic: $\div$

$\equiv$ is mod 26 for this slide.
$\frac{1}{3} \equiv x$ where $0 \le x \le 25$.
Pedantic: $\frac{1}{y}$ is the number such that $y \times \frac{1}{y} \equiv 1$.
$\frac{1}{3} \equiv 9$ since $9 \times 3 = 27 \equiv 1$.
Shortcut: there is an algorithm that finds $\frac{1}{y}$ quickly.
We will NOT study the algorithm later.

$\frac{1}{2} \equiv x$ where $0 \le x \le 25$.

Modular Arithmetic: $\div$

$\equiv$ is mod 26 for this slide.
$\frac{1}{3} \equiv x$ where $0 \leq x \leq 25$.
Pedantic: $\frac{1}{y}$ is the number such that $y \times \frac{1}{y} \equiv 1$.
$\frac{1}{3} \equiv 9$ since $9 \times 3 = 27 \equiv 1$.
Shortcut: there is an algorithm that finds $\frac{1}{y}$ quickly.
We will NOT study the algorithm later.

$\frac{1}{2} \equiv x$ where $0 \leq x \leq 25$. Think about it.

Modular Arithmetic: $\div$

$\equiv$ is mod 26 for this slide.
$\frac{1}{3} \equiv x$ where $0 \le x \le 25$.
Pedantic: $\frac{1}{y}$ is the number such that $y \times \frac{1}{y} \equiv 1$.
$\frac{1}{3} \equiv 9$ since $9 \times 3 = 27 \equiv 1$.
Shortcut: there is an algorithm that finds $\frac{1}{y}$ quickly.
We will NOT study the algorithm later.

$\frac{1}{2} \equiv x$ where $0 \le x \le 25$. Think about it.
No such x exists.

Modular Arithmetic: $\div$

$\equiv$ is mod 26 for this slide.
$\frac{1}{3} \equiv x$ where $0 \le x \le 25$.
Pedantic: $\frac{1}{y}$ is the number such that $y \times \frac{1}{y} \equiv 1$.
$\frac{1}{3} \equiv 9$ since $9 \times 3 = 27 \equiv 1$.
Shortcut: there is an algorithm that finds $\frac{1}{y}$ quickly.
We will NOT study the algorithm later.

$\frac{1}{2} \equiv x$ where $0 \le x \le 25$. Think about it.
No such x exists.
Fact: A number y has an inverse mod 26 if y and 26 have no common factors. Numbers that have an inverse mod 26:

$$\{1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23, 25\}$$

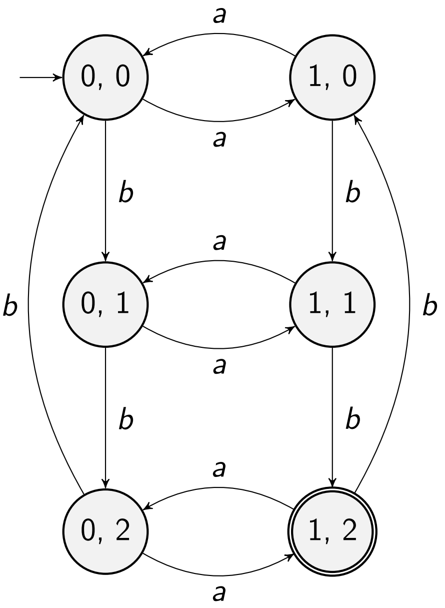
End of Detour

End of Detour

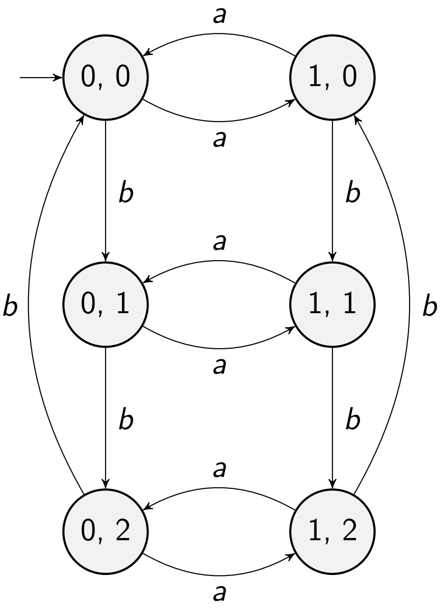
Another Example

$$\{w : \#_a(w) \equiv 1 \pmod 2 \wedge \#_b(w) \equiv 2 \pmod 3\}$$

$\{w : \#_a(w) \equiv 1 \pmod 2 \wedge \#_b(w) \equiv 2 \pmod 3\}$



$((\#_a(w) \pmod 2), \#_b(w) \pmod 3))$



$((\#_a(w) \pmod 2), \#_b(w) \pmod 3))$

A DFA-classifier does not ACCEPT and REJECT. It classifies.

$((\#_a(w) \pmod 2, \#_b(w) \pmod 3))$

A DFA-classifier does not ACCEPT and REJECT. It classifies.
If w is fed to the DFA in the last slide, the resulting state is

$$(\#_a(w) \pmod 2, \#_b(w) \pmod 3)$$

$((\#_a(w) \pmod 2), \#_b(w) \pmod 3))$

A DFA-classifier does not ACCEPT and REJECT. It classifies.
If w is fed to the DFA in the last slide, the resulting state is

$$(\#_a(w) \pmod 2), \#_b(w) \pmod 3))$$

The first DFA **accepted** $(1, 2)$-strings and **rejected** the rest.

$((\#_a(w) \pmod 2), \#_b(w) \pmod 3))$

A DFA-classifier does not ACCEPT and REJECT. It classifies.
If w is fed to the DFA in the last slide, the resulting state is

$$(\#_a(w) \pmod 2), \#_b(w) \pmod 3))$$

The first DFA **accepted** $(1, 2)$-strings and **rejected** the rest.

The second DFA **classifies** strings without judgment.

Short Detour

Short Detour

Alphabets, Strings, and Languages

Alphabets and Strings

Alphabets and Strings

Def An **alphabet** Σ is a set of letters (or characters).

Alphabets and Strings

Def An **alphabet** Σ is a set of letters (or characters).

- For Examples 1 and 2: $\Sigma = \{a, b\}$.

Alphabets and Strings

Def An **alphabet** Σ is a set of letters (or characters).

- For Examples 1 and 2: $\Sigma = \{a, b\}$.
- For Example 3: $\Sigma = \{$A, ..., Z, #, .$\}$.

Alphabets and Strings

Def An **alphabet** Σ is a set of letters (or characters).

- For Examples 1 and 2: $\Sigma = \{a, b\}$.
- For Example 3: $\Sigma = \{$A, ..., Z, $\#$, .$\}$.

Def A **string** or **word** is a sequence of symbols from an alphabet Σ.

Alphabets and Strings

Def An **alphabet** Σ is a set of letters (or characters).

- For Examples 1 and 2: $\Sigma = \{a, b\}$.
- For Example 3: $\Sigma = \{$A, ..., Z, #, .$\}$.

Def A **string** or **word** is a sequence of symbols from an alphabet Σ.

- $\Sigma^2 = \Sigma\Sigma = \{\sigma_1\sigma_2 : \sigma_1 \in \Sigma \wedge \sigma_2 \in \Sigma\}$.

Alphabets and Strings

Def An **alphabet** Σ is a set of letters (or characters).

- For Examples 1 and 2: $\Sigma = \{a, b\}$.
- For Example 3: $\Sigma = \{\text{A, ..., Z, \#, .}\}$.

Def A **string** or **word** is a sequence of symbols from an alphabet Σ.

- $\Sigma^2 = \Sigma\Sigma = \{\sigma_1\sigma_2 : \sigma_1 \in \Sigma \wedge \sigma_2 \in \Sigma\}$.
- $\Sigma^3 = \Sigma\Sigma\Sigma = \{\sigma_1\sigma_2\sigma_3 : \sigma_1 \in \Sigma \wedge \sigma_2 \in \Sigma \wedge \sigma_3 \in \Sigma\}$.

Alphabets and Strings

Def An **alphabet** Σ is a set of letters (or characters).

- For Examples 1 and 2: $\Sigma = \{a, b\}$.
- For Example 3: $\Sigma = \{$A, ..., Z, $\#$, .$\}$.

Def A **string** or **word** is a sequence of symbols from an alphabet Σ.

- $\Sigma^2 = \Sigma\Sigma = \{\sigma_1\sigma_2 : \sigma_1 \in \Sigma \wedge \sigma_2 \in \Sigma\}$.
- $\Sigma^3 = \Sigma\Sigma\Sigma = \{\sigma_1\sigma_2\sigma_3 : \sigma_1 \in \Sigma \wedge \sigma_2 \in \Sigma \wedge \sigma_3 \in \Sigma\}$.
- $\Sigma^i = \{\sigma_1 \cdots \sigma_i : \sigma_1, \ldots, \sigma_i \in \Sigma\}$

Alphabets and Strings

Def An **alphabet** Σ is a set of letters (or characters).

- For Examples 1 and 2: $\Sigma = \{a, b\}$.
- For Example 3: $\Sigma = \{$A, ..., Z, $\#$, .$\}$.

Def A **string** or **word** is a sequence of symbols from an alphabet Σ.

- $\Sigma^2 = \Sigma\Sigma = \{\sigma_1\sigma_2 : \sigma_1 \in \Sigma \wedge \sigma_2 \in \Sigma\}$.
- $\Sigma^3 = \Sigma\Sigma\Sigma = \{\sigma_1\sigma_2\sigma_3 : \sigma_1 \in \Sigma \wedge \sigma_2 \in \Sigma \wedge \sigma_3 \in \Sigma\}$.
- $\Sigma^i = \{\sigma_1 \cdots \sigma_i : \sigma_1, \ldots, \sigma_i \in \Sigma\}$
- $i = 1$ case is just $\Sigma^1 = \Sigma$.

Alphabets and Strings

Def An **alphabet** Σ is a set of letters (or characters).

- For Examples 1 and 2: $\Sigma = \{a, b\}$.
- For Example 3: $\Sigma = \{$A, ..., Z, #, .$\}$.

Def A **string** or **word** is a sequence of symbols from an alphabet Σ.

- $\Sigma^2 = \Sigma\Sigma = \{\sigma_1\sigma_2 : \sigma_1 \in \Sigma \wedge \sigma_2 \in \Sigma\}$.
- $\Sigma^3 = \Sigma\Sigma\Sigma = \{\sigma_1\sigma_2\sigma_3 : \sigma_1 \in \Sigma \wedge \sigma_2 \in \Sigma \wedge \sigma_3 \in \Sigma\}$.
- $\Sigma^i = \{\sigma_1 \cdots \sigma_i : \sigma_1, \ldots, \sigma_i \in \Sigma\}$
- $i = 1$ case is just $\Sigma^1 = \Sigma$.
- $i = 0$ case is just $\Sigma^0 = \{e\}$ **(the empty string)**.

Alphabets and Strings

Def An **alphabet** Σ is a set of letters (or characters).

- For Examples 1 and 2: $\Sigma = \{a, b\}$.
- For Example 3: $\Sigma = \{$A, ..., Z, $\#$, .$\}$.

Def A **string** or **word** is a sequence of symbols from an alphabet Σ.

- $\Sigma^2 = \Sigma\Sigma = \{\sigma_1\sigma_2 : \sigma_1 \in \Sigma \wedge \sigma_2 \in \Sigma\}$.
- $\Sigma^3 = \Sigma\Sigma\Sigma = \{\sigma_1\sigma_2\sigma_3 : \sigma_1 \in \Sigma \wedge \sigma_2 \in \Sigma \wedge \sigma_3 \in \Sigma\}$.
- $\Sigma^i = \{\sigma_1 \cdots \sigma_i : \sigma_1, \ldots, \sigma_i \in \Sigma\}$
- $i = 1$ case is just $\Sigma^1 = \Sigma$.
- $i = 0$ case is just $\Sigma^0 = \{e\}$ **(the empty string)**.
- **Notation Kleene star:** $\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \cdots$ is the set of all strings over the alphabet Σ (including e).

Languages

Languages

Def A **language** over an alphabet Σ is a subset of Σ^*.

Languages

Def A **language** over an alphabet Σ is a subset of Σ^*.

Def Let M be a machine that accepts (or rejects) words.
Then the **language** $L(M) = \{w : M \text{ accepts } w\}$.

Languages

Def A **language** over an alphabet Σ is a subset of Σ^*.

Def Let M be a machine that accepts (or rejects) words. Then the **language** $L(M) = \{w : M \text{ accepts } w\}$.

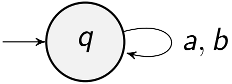
Draw the DFA that accepts the empty language over the alphabet $\{a, b\}$. I.e., $L = \{\}$.

Languages

Def A **language** over an alphabet Σ is a subset of Σ^*.

Def Let M be a machine that accepts (or rejects) words.
Then the **language** $L(M) = \{w : M \text{ accepts } w\}$.

Draw the DFA that accepts the empty language over the alphabet $\{a, b\}$. I.e., $L = \{\}$.

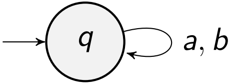


Languages

Def A **language** over an alphabet Σ is a subset of Σ^*.

Def Let M be a machine that accepts (or rejects) words.
Then the **language** $L(M) = \{w : M \text{ accepts } w\}$.

Draw the DFA that accepts the empty language over the alphabet $\{a, b\}$. I.e., $L = \{\}$.



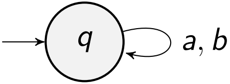
Draw the DFA that accepts the language L over the alphabet $\{a, b\}$ with only the empty word. I.e. $L = \{e\}$.

Languages

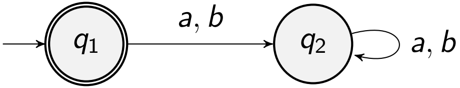
Def A **language** over an alphabet Σ is a subset of Σ^*.

Def Let M be a machine that accepts (or rejects) words.
Then the **language** $L(M) = \{w : M \text{ accepts } w\}$.

Draw the DFA that accepts the empty language over the alphabet $\{a, b\}$. I.e., $L = \{\}$.



Draw the DFA that accepts the language L over the alphabet $\{a, b\}$ with only the empty word. I.e. $L = \{e\}$.



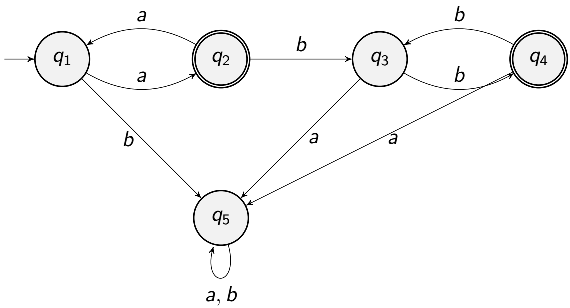
End of Detour

End of Detour

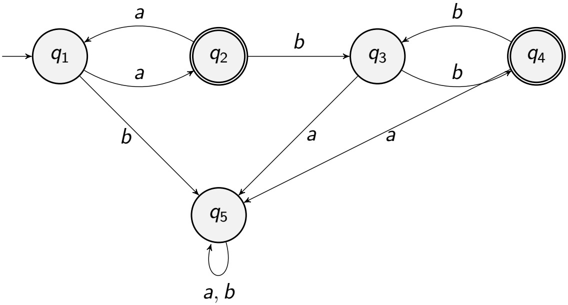
Start of Transition Tables

Recall Second Example

Recall Second Example

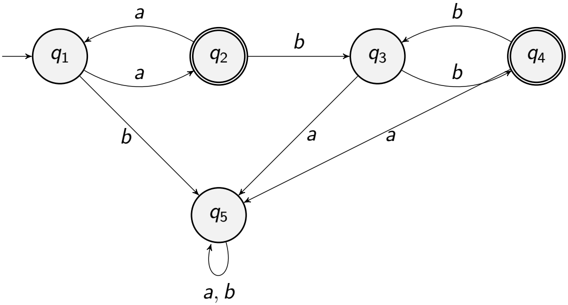


Recall Second Example



Transition Table:

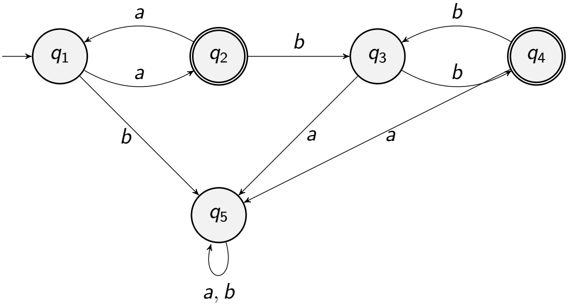
Recall Second Example



Transition Table:

- States: $\{q_1, q_2, q_3, q_4, q_5\}$

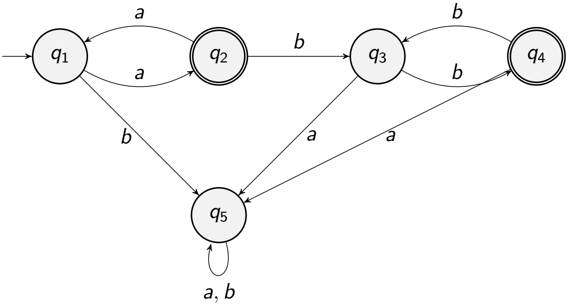
Recall Second Example



Transition Table:

- States: $\{q_1, q_2, q_3, q_4, q_5\}$
- Alphabet: $\{a, b\}$

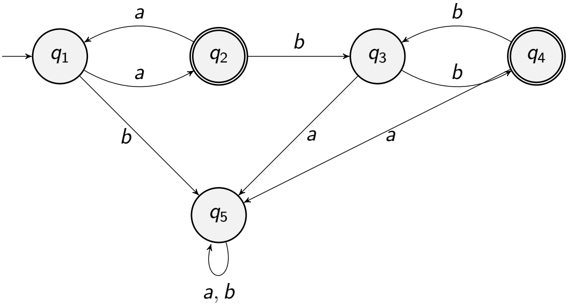
Recall Second Example



Transition Table:

- States: $\{q_1, q_2, q_3, q_4, q_5\}$
- Alphabet: $\{a, b\}$
- Start state: q_1

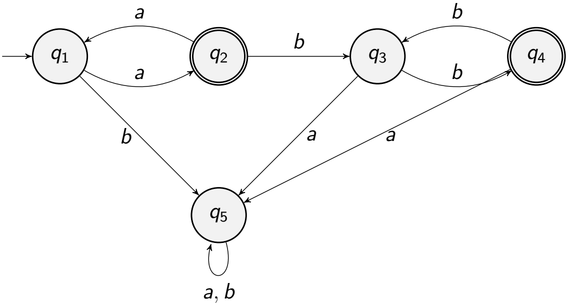
Recall Second Example



Transition Table:

- States: $\{q_1, q_2, q_3, q_4, q_5\}$
- Alphabet: $\{a, b\}$
- Start state: q_1
- Final states: $\{q_2, q_4\}$

Recall Second Example



Transition Table:

- States: $\{q_1, q_2, q_3, q_4, q_5\}$
- Alphabet: $\{a, b\}$
- Start state: q_1
- Final states: $\{q_2, q_4\}$

- Transition function

	a	b
q_1	q_2	q_5
q_2	q_1	q_3
q_3	q_5	q_4
q_4	q_5	q_3
q_5	q_5	q_5

Formal definition of DFAs

Formal definition of DFAs

Def A **DFA** M is a 5-tuple $(Q, \Sigma, \delta, s, F)$ where:

1. Q is a finite set of **states**.
2. Σ is a finite **alphabet**.
3. $\delta : Q \times \Sigma \to Q$ is the **transition function**.
4. $s \in Q$ is the **start state**.
5. $F \subseteq Q$ is the set of **final states**.

Formal definition of DFAs

Def A **DFA** M is a 5-tuple $(Q, \Sigma, \delta, s, F)$ where:

1. Q is a finite set of **states**.
2. Σ is a finite **alphabet**.
3. $\delta : Q \times \Sigma \to Q$ is the **transition function**.
4. $s \in Q$ is the **start state**.
5. $F \subseteq Q$ is the set of **final states**.

Informally DFA M accepts w if when M is run on w it ends up in a final state.

Formal definition of DFAs

Def A **DFA** M is a 5-tuple $(Q, \Sigma, \delta, s, F)$ where:

1. Q is a finite set of **states**.
2. Σ is a finite **alphabet**.
3. $\delta : Q \times \Sigma \to Q$ is the **transition function**.
4. $s \in Q$ is the **start state**.
5. $F \subseteq Q$ is the set of **final states**.

Informally DFA *M* accepts *w* if when M is run on w it ends up in a final state.

Formally
Def If M is a DFA and $w \in \Sigma^*$ is a word of length n, then ***M* accepts *w*** if there is a sequence of states $r_0, r_1, r_2, \ldots, r_n$ such that $r_0 = s$, $r_i = \delta(r_{i-1}, x_i)$ for $1 \le i \le n$, and $r_n \in F$.

Formal definition of DFAs

Def A **DFA** M is a 5-tuple $(Q, \Sigma, \delta, s, F)$ where:

1. Q is a finite set of **states**.
2. Σ is a finite **alphabet**.
3. $\delta : Q \times \Sigma \to Q$ is the **transition function**.
4. $s \in Q$ is the **start state**.
5. $F \subseteq Q$ is the set of **final states**.

Informally DFA M accepts w if when M is run on w it ends up in a final state.

Formally
Def If M is a DFA and $w \in \Sigma^*$ is a word of length n, then **M accepts w** if there is a sequence of states $r_0, r_1, r_2, \ldots, r_n$ such that $r_0 = s$, $r_i = \delta(r_{i-1}, x_i)$ for $1 \leq i \leq n$, and $r_n \in F$.

Def Language $L \subseteq \Sigma^*$ is **regular** if there exists a DFA M such that $L(M) = L$.

Computer Implementation of DFAs

Recall Second Example

Recall Second Example

Transition Table:

- States: $\{q_1, q_2, q_3, q_4, q_5\}$
- Alphabet: $\{a, b\}$
- Start state: q_1
- Final states: $\{q_2, q_4\}$

- Transition function

	a	b
q_1	q_2	q_5
q_2	q_1	q_3
q_3	q_5	q_4
q_4	q_5	q_3
q_5	q_5	q_5

Recall Second Example

Transition Table:

- States: $\{q_1, q_2, q_3, q_4, q_5\}$
- Alphabet: $\{a, b\}$
- Start state: q_1
- Final states: $\{q_2, q_4\}$

- Transition function

	a	b
q_1	q_2	q_5
q_2	q_1	q_3
q_3	q_5	q_4
q_4	q_5	q_3
q_5	q_5	q_5

Implementation of Transition Table:

- States: $\{1, 2, 3, 4, 5\}$
- Alphabet: $\{1, 2\}$
- Start state: 1
- Final states: $\{2, 4\}$

- Transition function

	1	2
1	2	5
2	1	3
3	5	4
4	5	3
5	5	5

Recall Second Example

Transition Table:

- States: $\{q_1, q_2, q_3, q_4, q_5\}$
- Alphabet: $\{a, b\}$
- Start state: q_1
- Final states: $\{q_2, q_4\}$

- Transition function

	a	b
q_1	q_2	q_5
q_2	q_1	q_3
q_3	q_5	q_4
q_4	q_5	q_3
q_5	q_5	q_5

Implementation of Transition Table:

- States: $\{1, 2, 3, 4, 5\}$
- Alphabet: $\{1, 2\}$
- Start state: 1
- Final states: $\{2, 4\}$

- Transition function

	1	2
1	2	5
2	1	3
3	5	4
4	5	3
5	5	5

Linear time!

Diagrams Versus Transition Tables

Diagrams Versus Transition Tables

Finite state automata are essentially graphs. Same rules apply:

Diagrams Versus Transition Tables

Finite state automata are essentially graphs. Same rules apply:

- Diagrams are good for people to understand if the DFAs are small.

Diagrams Versus Transition Tables

Finite state automata are essentially graphs. Same rules apply:

- Diagrams are good for people to understand if the DFAs are small.
- Transition tables are good for algorithms and formal proofs.