HW02 Solution

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Number of States: pq.

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$$F = \{(i_1, \ldots, i_k) : \text{ the numb of } i\text{'s that are 0 is a square } \}.$$

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Intuition $\exists$ path $p \rightarrow q$ in $M \implies \exists$ path $q \rightarrow p$ in M^R.

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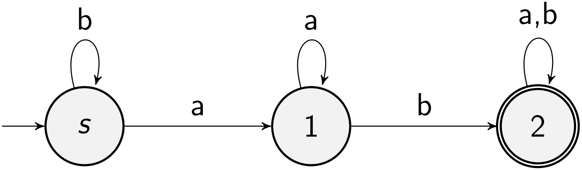
DFA for L:



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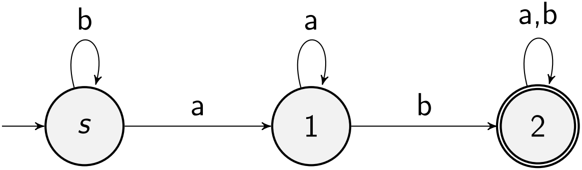
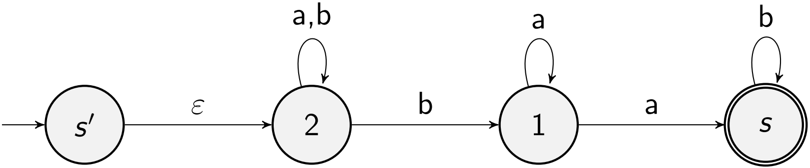


NFA for L^R:



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