HW07: Solution to Problem 7

NP in DTIME PROBLEM

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People did this three ways. I discuss all three.

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I do not like it since it uses Cook-Levin; however, it is correct.

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