HW09 Solution

ADAM SETS: PROBLEM

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1) Show that if A is Σ_1 then A is an ADAM set.
2) Show that if A is an ADAM set then $A \in \Sigma_1$.

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$$A = \{x : (\exists s)[M_{e,s}(x) \downarrow]\}.$$

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Show that if L is Σ_1 then $\mathrm{ISAAC}(L)$ is Σ_1.

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$$\mathrm{ISAAC}(L) = \{x : (\exists y, z)[x \in \mathrm{ISAAC}(z) \wedge (z, y) \in B].$$

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$x \in \mathrm{ISAAC}(L) \rightarrow (\exists y \in L)[x \in \mathrm{ISAAC}(y)] \rightarrow M(x) \downarrow$.

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$x \in \mathrm{ISAAC}(L) \rightarrow (\exists y \in L)[x \in \mathrm{ISAAC}(y)] \rightarrow M(x) \downarrow$.

$x \notin \mathrm{ISAAC}(L) \rightarrow \neg(\exists y \in L)[x \in \mathrm{ISAAC}(y)] \rightarrow M(x) \uparrow$.

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- If $L \in \mathrm{DEC}$ then $\mathrm{ISAAC}(L) \in \mathrm{DEC}$. **Fire** and **Brimstone**.

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A special case is on the next slide.

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Hence $L \subseteq a^*$ and $L \in \mathrm{DEC}$ implies $\mathrm{ISAAC}(L) \in \mathrm{DEC}$.

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Thm Let L be **any** language. $\exists\ w_1, \ldots, w_m$:
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Bizarre Thm If L is **any lang whatsoever** then $\mathrm{ISAAC}(L)$ is regular.

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4. **Key intuition** about why its true: the operation $\mathrm{ISAAC}(L)$ wipes out lots of information.
   Example $\mathrm{ISAAC}(a^n b^n c^n d^n) = a^* b^* c^* d^*$.

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If $C_4 = (x_{10} \vee \neg x_{11} \vee x_{12} \vee \neg x_{12})$ then **Replace this Clause for T**.