

1 Page 2

1. I agree that the addition to Notation 2.3 sufficiently addresses the comment that I made about "adjacent elements".
2. Looking at the Notation 2.3 section again also made me realize that the = in Notation 2.3.2 should be a minus sign instead.

YES. I HAVE MADE THE CORRECTION.

3. Also, if the elements of R, G, B are assumed to be in increasing order like in the original paper, we can drop the absolute value signs.

YES. I have also added $<$ in the definitions of the sets R,B,G.

4. We should add this assumption in this section too, since on page 4 (and other places) we use the fact that $z''_k < z'_k$ implies $k'' < k'$.

NOW THAT I'VE MADE IT EXPLICIT THAT $z_1 < z_2 < \dots < z_c$
DO I NEED TO DO ANYTHING FOR THIS CORRECTION?

2 Page 4

1. In the subcases for $z_k + r - y_1 \in G$, the first two cases should be $z_k - y_1$ instead of $z_k - y$.

YES. I HAVE MADE THE CORRECTION.

2. The RHS for the rainbow solution in the case $z_k - y_1 \in R$ should be z_k .

YES. I HAVE MADE THE CORRECTION.

3. (This is not a correction you gave me, its a thought on my part.)

The dashes that are used $z_k + r - y_1 \in G$ case look like negative signs.
So I made everying enumerate rather than some itemize, and now all the cases are indexed by numbers or letters.

3 Page 5

By -C2 $y_1 \leq r$ should have y_1 instead of y .

YES. I HAVE MADE THE CORRECTION.

4 Page 6

Commenting on disagreement, originally under page 4: Sure, makes sense, though the last sentence of your disagreement seems to be cut off.

I DON'T RECALL WHAT I WANTED TO SAY THERE; HOWEVER, IF YOU ARE HAPPY KEEPING ALL OF THE CASES, AND YOU SEEM TO BE, THEN THIS ISSUE IS SETTLED.

5 Page 8

1. Typo edit: Right under the line *Case 5*, it should be *We will look at the colors...*

YES. I HAVE MADE THE CORRECTION.

2. In the *Color of $z_k + 1$* section, the last bullet point should be $z_k + 1$.

YES. I HAVE MADE THE CORRECTION.

3. In the *Color of $z_k - 2$* section, we've already shown that $z_k - 1$ is R, so we know $z_{k-1} = z_k - 2$.

I DISAGREE BUT GIVEN YOUR TRACK RECORD I AM PROBABLY WRONG. HERE IS WHAT I SEE:

r is the least distance between adjacent elements of G .

k is the least number such that $z_{k+1} - z_k = r$.

On Page 8 we have $r = 2$ so $z_{k+1} = z_k + 2$.

So in a line we have

$z_k - 1(\text{R}), z_k(\text{G}), z_k + 1(\text{B}), z_{k+1} = z_k + 2(\text{G})$

All we know about z_{k-1} is that its at least 3 away from z_k since k was the least k with $z_{k+1} - z_k = r = 2$.

4. A typo edit: ... *this contradicts $r = 2$. If ther are 2 apart...*

The *ther* should be *they*.

YES. I HAVE MADE THE CORRECTION.

6 Page 10

1. Typo edit, right above *Base Case: The proof is by induction*

YES. I HAVE MADE THE CORRECTION.

2. OK, I can understand the use of this specific hypothesis. I think I was implicitly assuming strong induction in which case having both $2i - 1$ and $2i + 1$ in the hypothesis would be redundant, but I see now that you're using weak induction and specifically constructing the hypothesis to include the terms whose colors you're assuming in the inductive step. Since you tend to include every deduction, I think mentioning that

$2(i + 1) - 1 = 2i + 1$ is already assumed to be in R , so we just need to show that $2(i + 1) + 1 = 2i + 3$ is in R would make that a bit more clear.

I DO NOT KNOW WHAT YOU ARE ASKING ME TO ADD.

7 Page 11

1. 2nd bullet point of "Color of $2i + 1 - z_k$ " should be G instead of C

YES. I HAVE MADE THE CORRECTION.

2. In that same bullet point, $z_k - 2$ should be B.

YES. I HAVE MADE THE CORRECTION.

3. In the first bullet point of "Color of $2i + 3$ ", the colors of the rainbow solution should be (G, R, B) in order.

YES. I HAVE MADE THE CORRECTION.

8 Whats Next

The Schönheim paper *Partitions of the Positive Integers With No x, y, z Belonging to Distinct Classes Satisfying $x + y = z$* is a natural next paper. It's longer and uses terminology that modern papers no longer use. Section 4 in seems to be a natural extension giving a bound on the number of color classes that an "admissible" (rainbow-free) coloring of $[n]$ can have, which

leads into the modern study of rainbow numbers given an equation and a set.

9 Comments on what else attached

Attached the pdf/tex. We have many smaller lemmas that have been omitted, as well as some results that exist but do not build up to a complete "story" (namely, lots of lower bounds on rainbow numbers for various types of composite n , where we have no sense for how tight these bounds are). The most cohesive progress made was for $x + y = z^2$ in Z_p , with the only "holes" being the Fermat primes. I can include more detail on the omitted parts as desired.