Lemmas Needed For Finite Canonical Ramsey Theorem

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Definitions

Definition: Let $COL : \binom{[m]}{2} \rightarrow \omega$. Let c be a color and let $v \in [m]$.

1. $\deg_c(x)$ is the number of c-colored edges (x, y).
2. A *bad triple* is a triple a, b, c such that a, b, c does not form a rainbow K_3.

If Degree is Low: Rainbow!

Lemma: Let $COL : \binom{[m]}{2} \rightarrow \omega$ be such that, for every color c and vertex v, $\deg_c(v) \leq d$. Then there exists a Rainbow set of size $\Omega(m^{1/3})$.

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Attempt

Lemma: Let $COL : \binom{[m]}{2} \longrightarrow \omega$ be such that, for every color c and vertex v, $\deg_c(v) \leq d$. Then the number of bad triples is less than $\frac{dm^2}{6}$.

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Attempt

Lemma: Let $COL : \binom{[m]}{2} \to \omega$ be such that there are $\leq b$ bad triples. Let $1 \leq m' \leq m$. There exists an m'-sized set of vertices with $\leq b\left(\frac{m'}{m}\right)^3$ bad triples.

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