CMSC 858M: Algorithmic Lower Bounds
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Chapter 12: Counting Problems

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1 Overview

2 12.1

This seems like an excellent motivating chapter for what follows. It gives good setup for what a counting problem is and for why we care about this kind of problem. I think the examples could use a little work, such as the squares example, but otherwise it seems very good. I'm curious if Theorem 12.1.2 is an if and only if.

3 12.2

A good definition chapter. I found the term "parsimonious reduction" to be quite amusing. I think it would benefit from a brief explanation of the benefit of parsimonious reductions - something like "Note that if there is a parsimonious reduction from A to B and #A is #P-complete, then #B is also #P-complete."

4 12.3

The chapter seems fine. I'm starting to think that the term "parsimonious" is not playing well with semantic satiation, but otherwise it seems good.

5 12.4

The chapter seems mostly good. There isn't much to comment on.

6 12.5

This chapter is hard to judge without the addition of the figures which are missing. I think I would be more comfortable saying it was good if the figures were there to clarify the meaning of the proof and make the section less short.

7 12.6

This section is good, although very short.

8 12.7

This section is very interesting, and it's neat to see the original motivation for the counting problem presented. That said, this chapter is very disorganized and hard to follow. It is missing figures and references things before they are defined. This section's content is good, but it needs very heavy editing.

9 12.8

This section is very good, and I think the figure was perfect. Organizationally, I'd recommend not creating a new section for TH-POS-2SAT but instead having a brief introductory sentence. Otherwise, excellent section.

10 12.9

This is an excellent section, and had very interesting and novel content for me. I have no criticisms, but I am curious if ASP3COL is still in P if we consider colorings to instead be the set of isomorphic colorings.

11 Relevant Problems

- Counting Labeled Trees in a Graph - This was shown by Jerrum [1] to be #P-complete.
- Bipartite Vertex Cover - This was shown by Provan and Ball [2] to be #P-complete.
- Antichain - Given a set X with a partial order, output the number of antichains, or sets with all elements pairwise incomparable. This was shown by Provan and Ball [2] to be #P-complete.
- Maximum Cardinality Bipartite Vertex Cover - This was shown by Provan and Ball [2] to be #P-complete.

- Maximum Cardinality Bipartite Independent Set - This was shown by Provan and Ball [2] to be #P-complete.
- Maximum Cardinality Antichain - This was shown by Provan and Ball [2] to be #P-complete.
- Minimum Cardinality (s, t) Cut - Given a graph and two vertices s, t in V, the number of minimum-size edge cuts separating s and t. This was shown by Provan and Ball [2] to be #P-complete.
- Bipartite 2-SAT without Negations - Given two sets X, Y of variables and a boolean formula B of form $\bigwedge(x_i \vee y_j)$ with $x_i \in X, y_j \in Y$, the number of satisfying assignments. This was shown by Provan and Ball [2] to be #P-complete.
- Eulerian Circuits - Counting the number of cycles in a graph that use each edge exactly once was shown to be #P-complete by Brightwell and Winkler [4]
- Fillmat - Fillmat is a logic puzzle published by Nikoli. It was shown to be both NP- and ASP-complete by Uejima and Suzuki [3].

References

[1] Mark Jerrum (1994). Counting trees in a graph is #P-complete. Information Processing Letters, 51(3), 111-116.

[2] Provan, J., & Ball, M. (1983). The Complexity of Counting Cuts and of Computing the Probability that a Graph is Connected. SIAM Journal on Computing, 12(4), 777-788.

[3] Uejima, Akihiro & Suzuki, Hiroaki. (2015). Fillmat is NP-Complete and ASP-Complete. Journal of Information Processing. 23. 310-316. 10.2197/ipsjjip.23.310.

[4] Brightwell, G. R., & Winkler, P. (2005, January). Counting Eulerian Circuits is #P-Complete. In ALENEX/ANALCO (pp. 259-262).