1 Further Results

1.1 More APSP-Complete Problems

1. MINIMUM WEIGHT CYCLE IN GRAPH OF NON-NEGATIVE EDGE WEIGHT: Given a weighted graph with nonnegative edge weights, find the minimum weight cycle in the graph. Williams & Williams [2] showed that this problem is APSP-complete.

2. SECOND SHORTEST SIMPLE PATH is as follows: Given a weighted directed graph G, and two nodes s and t, find the second shortest simple path between s and t in G. Williams & Williams [2] showed that this problem is APSP-complete.

3. CODIAMETER: Given a graph G, the goal of CoDiameter is to report a vertex which does not participate in an edge of length equal to the diameter of G. Boroujeni et al. [1] showed that this problem is APSP-complete by a reduction from APSP. Boroujeni also defines CORADIUS, CORADIUS, CONEGATIVETRIANGLE, and COMEDIAN are APSP-complete, and show them APSP-complete.

4. APSPVERIFICATION: Given a graph G and a matrix D, determine if D is the correct distance matrix for G. That is, check that

$$(\forall(i,j) \in E)[D_{i,j} = dist_G(i,j)].$$

   It is known that this problem is APSP-complete.

1.2 Misc

1. BOOLEAN MATRIX MULTIPLICATION (BMM): If boolean matrix multiplication has a sub cubic combinatorial algorithm, then so does the triangle detection problem in graphs. This was shown by Williams & Williams [2]. All known algorithms for triangle detection take cubic time, hence using the hardness of triangle detection as an assumption is reasonable.

2. COAPSPVERIFICATION: Given a graph G and a matrix D, either find a pair (i,j) such that $D_{i,j}$ is equal to the distance between vertices i and j in G, or determine that there is no such pair. Boroujeni et al. [1] gave a subcubic reduction from DIAM to it. Recall that DIAM is thought to require cubic time; however, we do not know if DIAM is APSP-hard.

3. $\{-1,0,1\}-$APSP IS AS FOLLOWS: Given a weighted directed graph with edge weights in $\{-1,0,1\}$, compute the APSP. Despite the complication of having negative edge weights, this problem has a subcubic $(O(n^{2.52}))$ algorithm given by Zwick [3]. This problem seemed to require cubic time but did not. Consider that a cautionary note.

References

[1] M. Boroujeni, S. Dehghani, S. Ehsani, M. T. Hajiaghayi, and S. Seddighin. Subcubic equivalences between graph centrality measures and complementary problems, 2019.
`http://arxiv.org/abs/1905.08127`.

[2] V. V. Williams and R. R. Williams. Subcubic equivalences between path, matrix, and triangle problems. *Journal of the Association of Computing Machinery (JACM)*, 65(5):27:1–27:38, 2018.
`https://doi.org/10.1145/3186893`.

[3] U. Zwick. All pairs shortest paths using bridging sets and rectangular matrix multiplication. *Journal of the ACM*, 49(3):289–317, 2002.
`https://doi.org/10.1145/567112.567114`.