1 ADD TO BEGINNING

There are several good books on fixed parameter tractability. We mention three: Downey & Fellows [2], Niedermeier [5], and Flum and Grohe [3].

2 CHANGE

CHANGE Nondeterministic Turing Machine Acceptance:
a) Make it Short Nondeterministic Turing Machine Computation SNTM
b) Change from all 0's to x, and x is part of the input.
ALSO mention

Problem 2.1 *2-CNF SAT (*$\mathsf{2SAT}$*)*
INSTANCE A 2-CNF formula ϕ *and* $k \in \mathbb{N}$*.*
QUESTION Is there a satisfying assignment of ϕ *with exactly* k *variables set to true. This is called* an assignment of weight k.
NOTE When referring to the parameterized version, $\mathsf{2SAT}$ *is sometimes called* Weighted $\mathsf{2SAT}$*.*

ADD stuff about two equiv Defs of $W[1]$-hard.

3 ADD TO $W[1]$ SECTION

Mathieson and Szeider [4] showed the following:

Theorem 3.1 *The* CLIQ *problem restricted to regular graphs (all the vertices have the same degree) is* $\mathsf{W}[1]$*-complete.*

4 ADD TO $W[2]$ SECTION

Problem 4.1 *Hitting Set (*HS*)*
INSTANCE : A hypergraph $H = (V, E)$ *and a number* $k \in \mathbb{N}$*. (*k *will be the parameter.)*
QUESTION : Is there a set $V' \subseteq V$ *such that, for all* $e \in E$*, there exists* $v \in V' \cap E$*? The set* V' *is called a* Hitting Set.

Downey & Fellows [2] (See also [5]) showed the following:

Theorem 4.2 HS *is* $W[2]$*-complete.*

5 NOT SURE WHERE TO PUT IT

Problem 5.1 *Perfect Codes (*PC*)*

INSTANCE A graph $G = (V, E)$ *and* $k \in \mathbb{N}$.

QUESTION Is there a $V' \subseteq V$, $|V'| = k$, *such that, each vertex* $v \in V$, *there is exactly on* $v' \in V'$ *that is either* v *or adjacent to* v.

This problem has an interesting history. Downey & Fellows [2] showed the following:

Theorem 5.2

1. *(Downey & Fellows [2])* IS *is reducible to* PC, *hence* PC *is* $W[1]$*-hard.*

2. *(Downey & Fellows [2])* $\mathsf{PC} \in W[2]$.

3. *(Cesati [1])* PC *is reducible to* SNTM *and hence* $\mathsf{PC} \in W[1]$. *Therefore* PC *is* $W[1]$*-complete.*

Downey & Fellows conjectured that PC is intermediary: in $W[2] - W[1]$ but not $W[2]$-complete. Hence it was a surprise when Cesati showed PC is $W[1]$-complete. It was interesting that Cesati showed $\mathsf{PC} \in W[1]$ by a reduction.

Exercise 5.3 This exercise will show why the problem is called *Perfect Code.* If x, y are strings of the same length then $d(x, y)$ is the number of bits they differ on.

Let $G_n = (V, E)$ be the graph with $V = \{0, 1\}^n$ and $E = \{(x, y) \colon d(x, y) = 1\}$.

1. Show that G_3 has a hitting set of size 2.

2. Show that G_4 has a hitting set of size 8.

3. Find a function $f(n) < 2^n$ such that, for all n, G_n has a hitting set of size $f(n)$.

4. Alice wants to send $x \in \{0, 1\}^n$ to Bob. She sends it over a line which sometimes flips 1 bit but never more. Restrict the strings Alice may send so that Bob can tell if an error occurred, and if so, correct it.

5. Read the literature on error correcting code. (Warning: It is vast.)

References

[1] M. Cesati. The turing way to parameterized complexity. *Journal of Computing and System Science*, 67(4):654–685, 2003. https://doi.org/10.1016/S0022-0000(03)00073-4.

[2] R. G. Downey and M. R. Fellows. *Parameterized Complexity*. Monographs in Computer Science. Springer, 1999. https://doi.org/10.1007/978-1-4612-0515-9.

[3] J. Flum and M. Grohe. *Parameterized Complexity Theory*. Texts in Theoretical Computer Science. An EATCS Series. Springer, 2006.

[4] L. Mathieson and S. Szeider. The parameterized complexity of regular subgraph problems and generalizations. In J. Harland and P. Manyem, editors, *Theory of Computing 2008. Proc. Fourteenth Computing: The Australasian Theory Symposium (CATS 2008), Wollongong, NSW, Australia, January 22-25, 2008. Proceedings*, volume 77 of *CRPIT*, pages 79–86. Australian Computer Society, 2008. http://crpit.scem.westernsydney.edu.au/abstracts/CRPITV77Mathieson.html.

[5] R. Niedermeier. *Invitation to Fixed-Parameter Algorithms*. Oxford University Press, 2006. https://doi.org/10.1093/ACPROF:OSO/9780198566076.001.0001.