# 1 Exercises

DO ALL OF THE POBLEMS IN KARP

# 2 Further Reading

We present a list of results which can be viewed either as further reading (we provide references) or exercises.

**Def 2.1** Let $G = (V, E)$ be a graph. A *biclique* is a set of two disjoint sets $A, B \subseteq V$ such that, for all $a \in A$ and $b \in B$, $(a, b) \in E$. An *induced biclique* is a biclique where there are no edges between vertices of $A$ or vertices of $B$.

**Problem 2.2** *Maximum Edge Biclique and Variants*
*INSTANCE : A bipartite graph* $G = (A, B, E)$ *and number* $k \in N$.
*QUESTION : Is there a biclique with* $|A| = |B| = k$.
*QUESTION : Is there a biclique with* $|A| + |B| \geq k$.
*QUESTION : Is there a biclique with* $|A| \times |B| \geq k$.

**Theorem 2.3**

1. *(This is stated by Garey & Johnson [1], where it is called* Balanced Complete Bipartite Subgraph*, and proven by Johnson [2].) The question of finding a biclique with* $|A| = |B| = k$ *is* NP*-complete.*

2. *(This is folklore.) The question of finding a biclique with* $|A| = |B| \leq k$ *is* P*. (Hint: Use matching.)*

3. *(Peeters [3]) The question of finding a biclique with* $|A| \times |B| \geq k$ *is* NP*-complete.*

**Problem 2.4** *Edge Dominating Set (*EDS*)*
*INSTANCE : A graph* $G = (V, E)$ *and number* $k \in N$.
*QUESTION : Is there a set* $E' \subseteq E$, $|E'| = k$, *such that every edge* $e \notin E'$ *is adjacent to an edge in* $E'$.

**Theorem 2.5**

1. EDS *is* NP*-complete.*

2. *(Yannakakis & Gavril [4])* EDS *is* NP*-complete even when restricted to bipartite graphs of degree 3. (Yannakakis & Gavril [4])* EDS *is* NP*-complete even when restricted to planar graphs of degree 3.*

# References


[1] M. R. Garey and D. S. Johnson. *Computers and Intractability.* W. H. Freeman and Company, New York, 1979.

[2] D. S. Johnson. The NP-completeness column: An ongoing guide. *Journal of Algorithms*, 8(3):438–448, 1987. https://doi.org/10.1016/0196-6774(87)90021-6.

[3] R. Peeters. The maximum edge biclique problem is NP-complete. *Discrete Applied Mathematics*, 131(3):651–654, 2003. https://doi.org/10.1016/S0166-218X(03)00333-0.

[4] M. Yannakakis and F. Gavril. Edge domination sets in graphs. *SIAM Journal of Applied Mathematics*, 38(3):364–372, 1980.