**ADD to STREAMING**

# 1 Further Readings

## 1.1 Non-Graph Problem

We often refer to $\mathsf{R}$ or $\mathsf{R^d}$. Note that real numbers are infinite in length. For all such problems there is a parameter that bounds the length of precision; however, we still think of the input as elements of $\mathsf{R}$ or $\mathsf{R^d}$.

1. The approximate null vector problem: given $x_1, \ldots x_{d-1}$ vectors in $\mathsf{R^d}$ output a vector that is approximately orthogonal to all of them. Dagan et al. [4] show that this problem has an $\Omega(d^2)$ lower bound.

2. Clarkson & Woodruff [3] consider a variety of Numerical Linear Algebra problems in the Streaming Model. They provide upper and lower boudns on the space complexity of one-pass algorithms. In what follows, $A$ is an $n \times d$ matrix, $B$ is an $n \times d'$ matrix and $c = d + d'$ and the input is assumed to be integers of $O(\log(nc))$ bits or $O(\log(nd))$ bits.

   (a) For outputing a matrix $C$ such that $||A^T B - C|| \leq \epsilon ||A|| \cdot ||B||$, they show that $\Theta(c\epsilon^{-2} \log(nc))$ space is needed.

   (b) For $d' = 1$, i.e, when $B$ is a vector $b$, finding an $x$ such that $||Ax - b|| \leq (1 + \epsilon) \min_{x' \in \mathsf{R^d}} ||Ax' - b||$ requires $\Theta(d^2 \epsilon^{-1} \log(nd))$ space.

## 1.2 Graph Problems

As usual $n$ is the number of vertices in the graph.

1. The Gap Cycle Counting Problem: Let $k$ be small. A graph $G$ is streamed which is either a disjoint union of $\frac{n}{k}$ $k$-cycles or a disjoint union of $\frac{n}{2k}$ $2k$-cycles. Determine which is the case. Assadi [1] showed that any $p$-pass streaming algorithm requires $n^{1-1/k^{\Omega 1/p}}$ space.

2. Assadi et al. [2] show that two-pass graph streaming algorithm for the $s$-$t$ reachability problem for directed graphs requires space $n^{2-o(1)}$.

3. Goel et al. [5] consider the maximum matching problem. They show that any single pass algorithm cannot achieve better than 2/3 approximation. There have been improvements to the bound since this work and most recently, [6] showed a $\frac{1}{1+ln2}$ bound.

4. Assadi [1] consider approximating the maximum matching problem for two pass algorithms and show that any such algorithm has approximation ratio at least $1 - \Omega(\frac{\log RS(n)}{\log n})$ where $RS(n)$ denotes maximum number of disjoint induced matchings of size $\theta(n)$.

# References


[1] S. Assadi. A two-pass lower bound for semi-streaming maximum matching, 2021.
https://arxiv.org/abs/2108.07187.

[2] S. Assadi and R. Raz. Near-quadratic lower bounds for two-pass graph streaming algorithms. In S. Irani, editor, *61st IEEE Annual Symposium on Foundations of Computer Science, FOCS 2020, Durham, NC, USA, November 16-19, 2020*, pages 342–353. IEEE, 2020.
https://doi.org/10.1109/FOCS46700.2020.00040.

[3] K. L. Clarkson and D. P. Woodruff. Numerical linear algebra in the streaming model. In M. Mitzenmacher, editor, *Proceedings of the 41st Annual ACM Symposium on Theory of Computing, STOC 2009, Bethesda, MD, USA, May 31 - June 2, 2009*, pages 205–214. ACM, 2009.
https://doi.org/10.1145/1536414.1536445.

[4] Y. Dagan, G. Kur, and O. Shamir. Space lower bounds for linear prediction in the streaming model. In A. Beygelzimer and D. Hsu, editors, *Conference on Learning Theory, COLT 2019, 25-28 June 2019, Phoenix, AZ, USA*, volume 99 of *Proceedings of Machine Learning Research*, pages 929–954. PMLR, 2019.
http://proceedings.mlr.press/v99/dagan19b/dagan19b.pdf.

[5] A. Goel, M. Kapralov, and S. Khanna. On the communication and streaming complexity of maximum bipartite matching. In Y. Rabani, editor, *Proceedings of the Twenty-Third Annual ACM-SIAM Symposium on Discrete Algorithms, SODA 2012, Kyoto, Japan, January 17-19, 2012*,

pages 468–485. SIAM, 2012.
`https://doi.org/10.1137/1.9781611973099.41`.

[6] M. Kapralov. Space lower bounds for approximating maximum matching in the edge arrival model. In D. Marx, editor, *Proceedings of the 2021 ACM-SIAM Symposium on Discrete Algorithms, SODA 2021, Virtual Conference, January 10 - 13, 2021*, pages 1874–1893. SIAM, 2021.
`https://doi.org/10.1137/1.9781611976465.112`.