Problems from ExamA.pdf to Put in the Book

Exercise 0.1 Let $k \in \mathsf{N}$. The *Max Connectivity Problem (*MCC(k)*)* is as follows: Given a graph G and a number k, determine if G has a connected component of size $\geq k$. Show that any multi-pass streaming graph algorithm solving MCC requires $\Omega(n)$ space. Show that this still holds when the graphs are restricted to be forests.

Exercise 0.2 The *Directed Steiner Forest Problem (*DSFP is as follows: given a graph and a set of pairs of vertices, find the smallest sub forest (i.e., a union of trees) such that every pair is connected. If we restrict the graph to be planar we call this the PDSFP problem. If we restrict the number of pairs of vertices to be $\leq k$ then we call this the PDSFP(k) problem.

Assume ETH. Show that there exists a function f such that PDSFP(k) cannot be solved in $f(k)n^{o(k)}$ time. Give the high level ideas of your proof as well as the details of the proof.

Exercise 0.3 unique coverage (Set Cover except that every elements is covered by only one set). Give an $O(\log n)$ approximation algorithm. Prove that it is polylog hard under appropriate hardness assumption.