HW01. ALL SET TO GO

Exercise 0.1 For each of the following problems, either (I) show that the problem is in P by giving a polynomial-time algorithm or (II) show that the problem is NP-hard by reducing one of the following to it: (a) 3-Partition, (b) 3-Dimensional Matching, or (c) Numerical 3-Dimensional Matching.

1. Given a set of numbers $A = \{a_1, \cdots, a_{2n}\}$ that sum to $t \cdot n$, find a partition of A into n sets $S_1, \cdots, S_n$ of size 2 such that each set sums to t.

2. Given a set of numbers $A = \{a_1, \cdots, a_{2n}\}$ that sum to $t \cdot n$, find a partition of A into n sets $S_1, \cdots, S_n$ of any size such that each set sums to t.

3. Given a set of numbers $A = \{a_1, \cdots, a_{2n}\}$ and a sequence of target numbers $\langle t_1, \cdots, t_n \rangle$, find a partition of A into n sets $S_1, \cdots, S_n$ of size 2 such that for each $i \in \{1, \cdots, n\}$, the sum of the elements in S_i is t_i.

Exercise 0.2 Give a direct reduction from 3-Partition to Partition. *Hint* First reduce directly from 3-Partition to Subset-Sum, then modify the proof to work with Partition.

Exercise 0.3 The *connected bisection problem* is as follows. The input is a graph $G = (V, E)$ with n vertices. Determine if V can be partitioned into two sets, each of size $n/2$ such that each part induces a connected subgraph. Show that this problem is NP-hard.

Exercise 0.4 Prove that for not all-equal integers a, b, and c, (a, a^3), (b, b^3) and (c, c^3) are collinear if and only if $a + b + c = 0$.

Exercise 0.5 Let $f : \mathsf{N} \to \mathsf{R}$ be a function, with $f(n) \geq n$. Prove that NSPACE$(f(n)) \subseteq$ DSPACE$(f(n)^2)$ (hence NPSPACE = PSPACE).

Exercise 0.6 Give a sub-cubic reduction from Negative-Triangle to Median.