HW03

Exercise 0.1 Recall that the SC is the problem where, given $U = \{1, \ldots, n\}$, $S_1, \ldots, S_m \subseteq [U]$, and k, can k of the S_i's cover U.

Dinur at Steurer [**?**] (via PCP) proved that if there exists $\epsilon < 1$ such that SC has a $(1 - \epsilon)$ approximation algorithm then P = NP.

We study the following variant of SC which is called *Maximum Coverage*. The input is, as for SC, $U = \{1, \ldots, n\}$, $S_1, \ldots, S_m \subseteq [U]$, and k. But now we ask what is the maximum number of elements of U that k of the S_i's can cover. Note that this is a function problem.

Assume that there exists an $\epsilon > 0$ and a $(1 - 1/e - \epsilon)$ approximation for the Maximum Coverge Problem. From this show that SC $\in$ P, so P = NP.

Exercise 0.2 Prove there is a parameterized reduction from dominating set to set cover.

Exercise 0.3 Connected dominating set is a dominating set which induces a connected graph on vertices in the dominating set.

1. Prove there is a parameterized reduction from dominating set to connected dominating set.
2. Prove connected dominating set is in $W[2]$ by creating an instance of Weighted Circuit Satisfiability with weft two for it.
3. Prove that connected dominating set is $W[2]$-complete.

Exercise 0.4 The *Strongly Connected Steiner subgraph problem* is as follows. The input is a directed graph G, a set $K \subseteq V(G)$ of terminals, and an integer l. The goal is to find a strongly-connected subgraph of G with at most l vertices that contains every vertex of K. Prove that the strongly connected Steiner subgraph problem is $W[1]$-hard by a parameterized reduction from multi-colored clique.

Exercise 0.5 Let $k \geq 3$. The *Tree-diam(k) problem* is the problem of deciding whether the diameter of an input tree T is at least k. Prove that any single-pass streaming algorithm for this problem needs at least $\Omega(n)$ space.