The Book Review Column[1]
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In this column we review the following books. They all involve of Automata Theory; however, they are all rather advanced.

1. **The Classical Decision Problem** by Egon Börger, Erich Grädel and Yuri Gurevich. Reviewed by Dan A. Simovici. This book discusses decidable and undecidable fragments of first-order logic. This book unifies and simplifies many of the previous proofs in the literature. The proofs of decidability are particularly interesting since the original proofs of these results were difficult but have been simplified over time.

2. **Automata theory and its applications** by Bakhadyr Khoussainov and Anil Nerode. Reviewed by Lawrence S. Moss and Hans-Jörg Tiede. This is a book about the decidability of theories that stresses the role of automata. The types of automata include tree automata, infinite automata, and infinite tree automata.

3. **Automata, Logics, and Infinite Games** edited by Erich Grädel, Wolfgang Thomas, and Thomas Wilke. Reviewed by Lawrence S. Moss and Hans-Jörg Tiede This is a collection of articles on the topics in the title, intended to bring the reader up to speed on current research. The review of this item is combined with the prior one. The authors of the review take this opportunity to publicize another application of automata-theoretic ideas that is not in any of the books reviewed here.

4. **Automatic sequences: Theory, Applications, Generalizations** by Jean-Paul Allouche and Jeffrey Shallit. Reviewed by Jean Berstel. Automatic sequences are sequences of symbols recognized by finite automata. These are the central topic of the book.

Books I want Reviewed

If you want a FREE copy of one of these books in exchange for a review, then email me at gasarchcs.umd.edu

Reviews need to be in LaTeX, LaTeX2e, or Plaintext.

Books on Algorithms

1. *Computational Discrete Mathematics: Combinatorics and Graph Theory with Mathematica* by Pemmaraju and Skiena.

2. *Algorithms: Design Techniques and Analysis* by Alsuwaiyel.

3. *Computational Techniques of the Simplex Method* by Maros.

4. *Immunocomputing: Principles and Applications* by Tarakanov, Skormin, Sokolova.

5. *Computational Line Geometry* by Pottmann and Wallner.

Books on Cryptography

1. *Data Privacy and Security* by David Salomon.
2. *Block Error-Correcting Codes: A Computational Primer* by Xambo-Descamps.

Misc Books

1. *Verification of Reactive Systems: Formal Methods and Algorithms* by Schneider.
2. *Information Theory, Inference, and Learning Algorithms* by MacKay.
3. *Finite Automata* by Lawson.
4. *Logic for Learning* by Lloyd.
5. *Combinatorial Designs: Constructions and Analysis* by Stinson.
6. *Selected Papers on Discrete Math* by Donald Knuth.

Review[2] of

The Classical Decision Problem
Authors: Egon Börger, Erich Grädel and Yuri Gurevich

Series: Universitext
Springer-Verlag, 1997
Second printing, 2001
Softcover, x + 482 pages
Reviewer: Dan A. Simovici, (dsim@cs.umb.edu)

1 Overview

This book is dedicated to a comprehensive presentation of the classical decision problem of first-order logic. The centrality of the original decision problem (which can be stated equivalently as the satisfiability problem for formulas, the validity problem for sentences, or the provability of formulas in a sound and complete formal system) has been identified by the founders of mathematical logic. Subsequent developments in mathematical logic and theoretical computer science, especially the formalization of the notion of computable function by Gödel, Kleene, and Herbrand, yielded a negative answer to the classical decision problem and focused the attention of researchers towards the identification of fragments of first-order logic that are decidable or undecidable.

The introduction to the book contains a historical perspective of this research effort that is worth reading first and re-reading often while working through the book. The notion of reduction class that is a mainstay of the treatment of decidability is also introduced in this initial chapter. Löwenheim's decidability result for formulas with unary predicates and his identification of the class of formulas with binary predicates as a reduction class are recalled as well as the work of Herbrand, Skolem, and Bernays.

Traditional fragments of first-order logic (considered as classes of formulas in prenex normal form that are defined by restrictions on the quantifier prefix or alphabet) are considered in detail for their decidability or undecidability; ample historical references are provided.

2 Summary of Contents

The book is divided into two parts, "Undecidable Classes" and "Decidable Classes and Their Complexity", comprising eight chapters. These parts are followed by an Appendix "Tiling Problems".

Part I: Undecidable Classes

contains chapters entitled "Reductions", "Undecidable Standard Classes for Pure Predicate Logic", "Undecidable Standard Classes with Functions and Equality", and "Other Undecidable Classes".

Chapter 2: Reductions introduces the central notion of reduction class for satisfiability as a class of formulas X such that there exists a computable function f that maps every satisfiable formula ϕ into a formula $f(\phi) \in X$.

The first section entitled "Undecidability and Conservative Reductions" begins with the the Church-Turing theorem on the undecidability of the classical decision problem. The argument is classical, presented in a pithy manner and consists in reducing the halting problem for deterministic Turing machines to the Entscheidungsproblem. The presentation is sufficiently general to allow specializations of the argument for several types of logics.

The same section deals with Skolem normal form theorem, Herbrand structures, Horn and Krom formulae (the latter defined as first-order formulas in prenex normal form whose quantifier-free part is a conjunction of clauses that contain at most two constituents). The Aanderaa-Börger reduction result for 2-register machines is presented here.

Trahtenbrot's result on the recursive inseparability of the sets of all finitely satisfiable, all unsatisfiable and all only infinitely-satisfiable formulas is presented in detail. Conservative reductions and conservative reduction classes are introduced as instruments that sometime simplify reduction proofs. This section concludes with a discussion of the relationship between inseparability and model complexity

The second section of this chapter, "Logic and Complexity" discusses the use of the logical specification of computations for the study of time-restricted or space-restricted hating problems in syntactically defined classes of formulas with solvable decision problems.

Cook-Levin Theorem on the NP-completeness of satisfiability for formulas of propositional logic is presented here, as well as the PSPACE-completeness of the decision problem for quantified propositional logic.

The notions of spectrum of a formula and generalized spectrum (as the class of a sentence in existential second-order logic) lead to a presentation of on Fagin's Theorem on generalized spectra.

Descriptive complexity theory is discussed starting from the the notion of a logic capturing a complexity class on a class of finite structures. Model-theoretic characterizations of P and NLOGSPACE on ordered structures are given. The final section of this chapter contains Gurevich's classifiability theorem that is an organizing principle of this area. Using the fact that the classes of prenex formulas form a well-order poset, where the collection of decidable classes is closed downward, it follows that there exist a finite number of minimal undecidable classes which dominates all undecidable classes, and there is a finite number of maximal decidable classes. Specifically, there are sixteen minimal undecidable classes and seven maximal decidable ones.

Chapter 3: Undecidable Standard Classes for Pure Predicate Logic This chapter is the part I enjoyed most. Building on Gurevich Classifiability Theorem, the main result of the chapter shows that a prefix-class of formulas without function symbols or equality is undecidable if it contains one of nine special classes of formulas. An inspired diagram placed in the introduction of the chapter summarizes various reductions between the classes involved and helps the reader get a clear picture of the aims of this chapter.

The first section entitled "The Kahr Class" ($[\forall\exists\forall, (\omega, 1)]$ in the notation of the authors) begins with the undecidability of domino problems introduced by Wang; actually, a stronger result on the recursive inseparability of the sets of domino problems that admit no tiling and those who admit a periodic tiling of $Z \times Z$ or $N \times N$. A new proof of this result is contained in a special appendix. The domino problem is used to prove that the Kahr class is a conservative reduction class. In turn, Kahr class plays a crucial role in proving the conservative reduction property of the other classes of interests. The rest of the chapter proves the conservative reduction properties for other classes of formulas.

Chapter 4: Undecidable Standard Classes with Functions or Equality contain results that establish the minimal undecidable prefix vocabulary classes of the full first-order logic, that is, formulas with relation symbols, function symbols, and equality. Supplementing the list of reduction classes of pure predicate logic, the list of classes established in this chapter (together with the decidability results established in chapters 6 and 7) gives a complete classification of prefix classes in the full first-order logic with respect to the decidability of the satisfiability.

Chapter 5: Other Undecidable Classes examines classes that are defined not only by the alphabet or prefix structure but also by the syntax of the quantifier-free part.

The undecidability of several prefix classes of Krom formulae without functions or equality is established (which is preserved even when restricted to Horn formulae) and conservative reduction classes are identified among the Krom and Horn formulae with function symbols or equality. Undecidability results are obtained for classes of formulas that have only a small number of atomic subformulae (important for logic programming). An example of the type of results presented here is the proof of Wirsing's result that the class of universal formulas containing only one equality and one inequality is a conservative reduction class. Other classes discussed in this chapter include formulas with a limited number of variables and conjunctions of prefix-vocabulary classes.

Part II: Decidable Classes and Their Complexity

Chapter 6: Standard Classes with Finite Model Property deals with a description of the standard classes of formulas for which the satisfiability and the finite satisfiability problems are decidable. The main result is a complete description of the seven maximal decidable classes. The decidability of five of these classes (which have the finite model property) is discussed in this chapter.

After introducing some novel techniques for proving complexity results (e.g. a new tiling problem on a torus) the decidability of several classical classes is shown. There classes were shown decidable before the undecidability of the general problem was shown by Church and Turing. The chapter concludes with a description of standard classes of "modest complexity", that is of prefix classes whose decision problems are in P, NP, or co-NP.

Chapter 7: Monadic Theories and Decidable Standard Classes with Infinity Axioms discusses the decidability of the remaining two maximal decidable classes (that is, of first-order logic with equality, one unary function symbol and monadic predicates, and the Shelah class that consists of formulas with at most one universal quantifier, at most one function symbol and arbitrary relation symbols, with equality). The proof of the decidability of the first class, due to one of the authors, is a simplification of Rabin's argument. It uses Games and Tree Automata. The argument for the Shelah class is an expansion of the original paper where this class was introduced.

Chapter 8: Other Decidable Classes is reserved for several special decidable classes that are important to computer scientists working in databases, logic programming, etc. An improvement of Mortimer's result for formulas with two variables is given.

In a section dedicated to unification such result as the log-space completeness for P of the unification problem, the P-completeness of the satisfiability for Herbrand formulas or the NP-completeness of the validity for the positive fragment of first-order logic are shown. The final section of this chapter deals with decidable classes of Krom formulas; various decidability results (e.g., the decidability of the Aanderaa-Lewis class) and techniques are presented.

3 Opinion

This book is an essential reference for any researcher in logic, complexity, and artificial intelligence.

The annotated bibliography of 549 titles is an enormous help for every researcher interested in decidability; it contains in a very concentrated form an enormous survey of the literature on the classical decision problem and enhances the role of the books as a reference source. Historical references that are placed at the end of each chapter are very enjoyable and help the reader follow the literature and gain a perspective of the field.

Occasional minor lapses, such as the inversion of the direction of reduction on page in the second paragraph on p. 29 can be easily spotted and do not diminish the value of this book.

In the preface the authors stress the efforts made to "combine the features of a research monograph and a textbook". It is my opinion that they succeeded in producing an excellent reference book for researchers in the field, and for advanced doctoral students in theoretical computer science and logic. However, I have my reservations on its use as a textbook. The material requires a lot more mathematical sophistication than the "basic knowledge of the language of first-order logic", as the authors claim. The presentation is very terse, with complex details left to a multitude of exercises (some very challenging) sprinkled throughout the text. In fairness to the authors, it seems to me that in view of the vast material included in this book, no other approach was realistic.

Review[3]
Automata theory and its applications
Authors: Bakhadyr Khoussainov and Anil Nerode
Progress in Computer Science and Applied Logic, 21.
Birkhäuser Boston, Inc., Boston, MA
2001. xiv+430 pp. $69.95. ISBN 0-8176-4207-2

And

Automata, Logics, and Infinite Games
Edited by E. Grädel, W. Thomas, and T. Wilke
Springer-Verlag, LNCS 2500
385 pp. ISBN 3-540-00388-6.

Reviews by
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1 Introduction

Although basic automata theory has spawned lots of books, and is a mainstay of theoretical and even practical computer science, advanced parts of the subject have not as yet had a strong impact on the computer science curriculum. Some of them have had applications, especially in model checking and verification. The books under review can serve as textbooks in formal foundations of model checking and verification, and at the same time they are texts in advanced automata theory.

Automata theory has been generalized in many ways: from finite state to various forms of memory, from deterministic to non-deterministic and then alternating, from non-stochastic to probabilistic, from discrete time to hybrid, etc. In the books under review, we are mainly concerned with the generalization of automata on finite inputs to automata on *infinite* words, and from words to *trees.* The main results of the subject still are the decidability of the monadic second-order logic of successor functions (S1S, S2S) due to Büchi and Rabin in the 1960's. These have been extended, simplified, and applied in numerous ways, and these are the core areas for the subject of this review.

Other topics in advanced automata theory involve combinatoric and algebraic ideas, but the books under review are concerned with logic and games. The areas of logic of special interest are second order logic and to some extent systems like the modal μ-calculus. The games involved are two-player games of perfect information, and these have had application in many areas of logic besides automata theory. While the subject is not of very great interest to game theorists, automata theory is of interest to *applied logicians*, since automata-theoretic methods wind up in solutions to purely logical problems such as decidability questions.

Finding the sources of this area usually involves many handbook articles or primary sources, there has at yet been no textbook-like treatments of either the "classical" or current literature. Even more, we know of no textbooks that contain material on infinite words and connections to second-order logic even as chapters "at the end". Computability theory and complexity theory seem to be the topics of choice for advanced treatments. We might also note the ACM curricular recommendations; they specify a minimal amount of automata theory but even as electives, and they do not mention advanced automata theory. However, the first book under review, opines that "They should all be part of every computer scientist's toolbox." (!)

In addition to the two books under review, there is a forthcoming book covering similar topics [8].

2 Review of "Automata theory and its applications"

We refer to the book as "KN" which are the initials of the authors.

This book is mainly an exposition of the decidability of S1S and S2S, along with the theory of ω-languages and the game theoretic treatment of S2S going back go Gurevich and Harrington [5]. Overall, the entire KN book may be read as an exposition of the papers by Büchi, McNaughton, and Rabin. Thus, it has the feeling of a retrospective book rather than a book about current work.

The first two chapters contain introductory material on basic set theory and also some material on finite automata theory. It purports to be a treatment from scratch, but it certainly could not be used as such. The book takes off in Chapter 3 on ω-automata. Here the treatment is slightly on the algebraic side. Perhaps it would have been nicer to see the shorter Ramsey-theoretic arguments in McNaughton's Theorem. Chapter 4 is on infinite games on finite graphs, following McNaughton's work on the subject. It must be noted that Chapter 4 is not used or mentioned later in the book; this must surely be one of the few cases in a mathematical book where a chapter in the middle could simply be skipped without losing anything. KN also do not really credit Chapter 4 to McNaughton in the text (they do in the introduction), and it is not exactly clear why they have chosen to include that chapter in the first place. It helps a bit to have seen Chapter 4 in Chapter 5, but this is not strictly necessary. Chapter 5 itself is on automata on infinite trees, and the game formulation of acceptance. It also contains the restricted memory determinacy theorem for Gurevich-Harrington games. The second main focus of the book is Chapter 6 on Rabin's Theorem on S2S, with a set of applications. Those applications are all from Rabin's papers, in particular they are discussed briefly in Rabin's 1969 paper [9].

The title of the book is somewhat misleading, since the applications are never treated. Indeed, when applied topics are used for motivation, the effect is not so good. (For example, the "applied" introduction to Chapter 3 seems spurious.)

Another criticism: The references/bibliography are a disaster. There are no notes at the end of chapters regarding sources or further material. Arranging the bibliography into sections without any comments makes it impossible to find anything. There are fairly basic references missing (although this is hard to verify given that they divided the references into sections). For instance, classical handbook articles by Gurevich [6], Rabin [10] and Thomas [14] are missing. Although the authors point out that fairly extensive bibliographies exist on the web, none of those are listed either.

In terms of coverage, alternating finite automata and Kleene algebra are not discussed, although both topics would fit well into the context of this book.

In addition, there are places where the material was not proofread at all: sections with lots of typos, an unusual aspect of the notation that the word **rank** is boldface for no reason throughout Chapter 5, etc. The exercises could definitely be improved, too: probably half of them simply ask the reader to prove a result used in the main body of the text.

Several discussions seemed to be repeated almost verbatim, as if the book were written at different times without a look back. For instance, there are two monadic second order logic sections in which a number of proofs appear in full detail in the same way.

On the positive side, it does do all the work about S2S in complete detail. For the most part, the discussions work well didactically. This is especially true of the Büchi automata chapter and the one on games on finite graphs.

Experience with teaching it: one of us gave an advanced graduate course using this book. Most of the students were mathematics students with little prior exposure to automata theory, and slightly fewer were advanced computer science theory students. The book worked fairly well as a textbook. Certainly Chapters 3 and 4 worked very well. Chapter 5 was much harder going. The determinacy result for games on trees is done in full detail in this book. This is extremely helpful for some people; but for others, it means that an instructor has to work to show the forest from the trees. In addition, an instructor would probably be wise to skip Chapter 4 (McNaughton's work on Games Played on Finite Graphs), since otherwise a semester might end without much said about Chapter 6 (Applications of Rabin Automata). However, some of the applications that are in Chapter 6 are definitely of greater interest to mathematicians than computer scientists. These include work on the monadic theory of countable linear orders and of the Cantor set. As we mentioned above, many people who are interested in advanced automata theory for teaching would probably prefer to look at other kinds of applications.

3 Review of "Automata, Logics, and Infinite Games"

We refer to this book as ALIG which are the initials of the editors.

This book aims to be a "Guide to Current Research." This means that it covers a lot of topics not known to the "classical" researchers of earlier days, but it also has a lot of the classical material, sometimes with newer proofs. For example, it goes through Klarlund's alternating automata to prove complementation of Büchi automata, and also Safra's results on the complexity of translations between ω-language formats. In a sense, it is a concise and uniform presentation of at least 50 papers, along with notes in various places that help the reader to put things together. It has a good bibliography (not as good as Börger et al. [1] for example, which covers similar material and is annotated).

The scope of the book is wider than KN, covering not only the ω-languages and MSO logic but also the modal μ-calculus and guarded logics, and tree-like models of MSO logic. The guarded fragment of first-order logic was invented by Johan van Benthem, and its decidability is a stunning success of automata-theoretic methods due to Grädel. This is the only textbook treatment of this material. Although some material on the μ-calculus is covered in books by Harel et al. [7] and also by Stirling [12]. It shows that the model checking problem for this logic is in UP $\cap$ co-UP; this is the current best result. It covers advanced issues on games not covered in KN such as the connection to alternating tree automata. It has a fuller discussion of memory requirements for infinite games and on complexity issues.

The book has the unusual feature of having 19 separate authors for its chapters. Described as "young researchers" they seem to be either post-docs or graduate students, most from Germany. The editors evidently did a good job in coordinating the contributions. For the most part, the material is better-integrated than one would expect. The writing is for the most part quite good, and it is only at rare points that one is conscious that the authors are not English speakers. It has plenty of examples and diagrams. Some of the chapters are foundational for the rest of the book, but the majority can be read separately. It is more of a handbook than most Handbooks.

We feel that the book could be used as a textbook for an advanced graduate course or seminar. Certainly it was not intended for use at lower levels (unlike KN, but that book also is not useful at lower levels). The ALIG book has useful notes at the beginning and ending of the different parts. It would be a good choice for a seminar involving students. For a book on current research, it would have been good to have a long list of open problems at the end (we think of Chang & Keisler's book on Model Theory [2]; one of theirs is to "develop a model theory for second order logic"!) At the end of this book we find an appendix on fixed-point results for partial orders. At six terse pages, this is not a good addition to the book; if one did not know about fixed points already the book's treatment would not help much.

Neither book covers a topic that we want to discuss momentarily, automata and logic on finite trees. Here there is the web-available TATA book [3]. The main results that we wish to mention are that regular languages of finite trees always have context-free string languages as their yields. Furthermore, a set of finite trees is regular iff it is definable in the MSO logic. This generalization of Büchi's Theorem was first obtained by Thatcher and Wright [13]; also by Doner [4].

Model theoretic syntax: an unexpected application We would like to publicize for readers of *SIGACT NEWS* a development that we are enthusiastic about concerning an application of automata-theoretic ideas. About 10 years ago, James Rogers connected the MSO logic of trees with grammar formalisms of then-current interest in linguistics and natural language processing. This work is described in his PhD and in his later book[11]. These formalisms used *well-formedness conditions* on trees as opposed to the more generative devices that we find in formal grammars. So in a sense, they are well-suited to the logical view. Grammaticality turns in to satisfiability. This opens the door to logical methods. Rogers observed all of this, and he also worked out the details of the connections between MSO logic and formalisms like Chomsky's Government and Binding Theory (GB) as well as Generalized Phrase Structure Grammar (GPSG). Since then, a number of researchers in mathematical linguistics have pursued extensions to this program.

The main problem with well-formedness conditions is that they are often presented informally, but this means that one cannot obtain any complexity measure on what they are defining, or even any decidability result. One cannot compare frameworks either. But having expressed the well-formedness in a logic like MSO, we have automatically given it a complexity measure. Furthermore, showing that something cannot be expressed in MSO on trees allows us to draw inferences about the complexity of a grammatical framework as well.

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Review[4] of
Automatic sequences: Theory, Applications, Generalizations
Authors: Jean-Paul Allouche and Jeffrey Shallit
Cambridge University Press
Author of Review: Jean Berstel, Institut Gaspard Monge, Université de Marne-la-Vallée

1 Overview

Automatic sequences are sequences of symbols recognized by finite automata. These are the central topic of the book.

Sequences of numbers or words occur in mathematics and in theoretical computer sciences, and also in theoretical physics, and in computational biology. This scattered research on more or less the same object explains widespread publication support, many variations in terminology, and a great variety of results.

Automatic sequences are simple to generate or to recognize, and moreover — and this is the main motivation of their study by number theorists — there are many interesting connections with number theory.

A famous example is the so-called Thue–Morse sequence t that reads, written on the digits 0 and 1 as

$$t = 0110100110010110\cdots$$

It can be defined in various ways. The simplest is by induction: $t_0 = 0$, and $t_{2n} = t_n$, $t_{2n+1} = 1 - t_n$. Another definition is to consider the binary expansion $\mathrm{bin}(n)$ of the integer n. Then $t_n = 1$ iff $\mathrm{bin}(n)$ contains an odd number of 1's, and $t_n = 0$ otherwise. This definition is "automatic" because one can easily conceive a two state automaton that recognizes those binary expansions that have an odd number of 1's. Another way to construct this infinite word t is to observe that it is a fixed point of the morphism μ that maps 0 to 01 and 1 to 10. Thus t can also be constructed as limit of a sequence of finite words, the so-called Thue–Morse blocks X_i defined by $X_0 = 0$ and $X_{i+1} = X_i\bar{X}_i$ where $\bar{w}$ exchanges 0 and 1 in w. There are several other magic ways to obtain t. One basic question of interest to number theorists is to determine the status of the real number with binary expansion $0.0110100110010110\cdots$.

It has been proved that this number is transcendental. As a matter of fact, there is a very close, amazing relationship between automatic sequences and transcendental numbers that is treated in detail in chapters 12 and 13 of the book. Basically, these numbers are either rational or transcendental although no general result of this kind has been proved. Another question is about the distribution of digits, and more generally of blocks, in this sequence. This is closely related to symbolic dynamics (this was the initial goal of Morse), but also to the search of so-called "normal numbers". These questions also are of interest in theoretical physics. Finally, it is well-known that the Thue–Morse sequence is cube-free (no three adjacent equal blocks) and even overlap-free (no overlapping equal blocks). This was the the domain of interest of Axel Thue. So Thue-Morse sequences are related to combinatorics on words, and to what biologists call "tandem repetitions".

The objective of the present book is to present automatic sequences and related fields under three point of views: combinatorics on words, especially in relation with automata theory, number theory, especially in relation with algebraic and transcendental numbers, functions, series, and applications to theoretical physics. It is the first time that such a presentation is given in a unified framework, using consistent notation and definition.

Recently, two books appeared which smell similar. First, Lothaire's second volume entitled "Combinatorics on Words" (Cambridge University Press 2002), and second Pytheas Fogg's "Substitutions in Dynamics, Arithmetics and Combinatorics" (Lecture Notes in Mathematics 1794, Springer-Verlag). In fact, Lothaire's book does not cover any relation to number theory with the exception of number systems, and Pytheas Fogg deals more with ergodic theory, spectral analysis and geometric representation of sequences. None of these books contains for instance a proof of Cobham's theorem. The nature of the book under review is much close to a textbook, and can be used so.

2 Contents

The book consists of 17 chapters and an Appendix. Although it is not structured explicitly in this way, it can be roughly viewed as composed of three parts: Chapters 1–4 are preliminaries, even if they may contain deep results. Chapters 5–13 are on automatic sequences, their generalizations and their applications, and chapters 14–17 are on generalizations and applications.

Chapter 1, **Stringology** introduces finite and infinite words, languages as sets of words, morphisms as a way to transform words. It also contains basic results on equations, such as Fine and Wilf's theorem.

The second part of this chapter contains the proof that the Thue-Morse word is overlap-free, and also detailed results on the overlap-free words that are fixed point of a morphism, and the characterization of overlap-free binary morphisms, that is morphisms that map overlap-free words on overlap-free words. This chapter is quite combinatoric in nature.

Chapter 2, **Number Theory and Algebra** is presented as a smorgasbord of some basic results from algebra and number theory. It presents a proof that π is irrational, and also Liouville's example of an irrational number. Some results on transcendental numbers are stated here without proof. Continued fractions and approximations are considered, with a proof that $\{n\theta\}$ is uniformly distributed for irrational θ. It also contains the three–distance theorem.

A second part deals with definition on algebraic structures, fields, polynomials, formal power series and formal Laurent series, p-adic numbers. Some estimates are grouped at the end of this chapter.

Chapter 3, **Numeration Systems** is on representation of numbers, mainly of integers, in several bases, first the standard integral base, then negative bases, and then Fibonacci representation and representation in complex bases. There are developments on the sum-of-digits function, and on pattern sequences, and on sums of sums of digits.

Fibonacci's number system is presented, and replaced in the context of Ostrowski's numeration system. There is a careful presentation of complex bases, and of perfect complex number systems (those with base $-a \pm i$ for some integer $a \geq 1$).

Chapter 4, **Finite Automata and Other Models of Computation** introduces finite automata, deterministic and nondeterministic, proves Kleene's theorem and Myhill–Nerode's theorem, a pumping lemma. It also considers finite automata with output on states: the result of a path is the output associated to the state (this corresponds to automata with several types of final states).

Transducers are introduced next, and closure of regular languages under transducer mapping is proved, and applications to sets of numbers recognized by finite automata are given.

Next, there is a short section on context-free grammars and languages, pda's and algebraic formal power series and on Turing machines.

Chapter 5, **Automatic Sequences** introduces the fundamental concept of the book. Automatic sequences are defined by automata with output on states. Many examples are given. Two-sided automatic sequences are reduced to one-sided sequences. The chapter contains some basic properties, also closure properties, and examples of non-automatic sequences.

Chapter 6, **Uniform Morphisms and Automatic Sequences** presents the second definition of automatic sequences, by uniform morphisms (Cobham's first theorem). The tower of Hanoi sequence is an excellent example treated in detail. Paper folding is considered. The kernel is introduced next (that is right regular equivalence on automata). Other closure properties are given, using either the kernel or transducers.

Chapter 7, **Morphic Sequences** presents the general theory of morphic sequences, moving thus from the uniform to the general case. A typical example is the Fibonacci word. A description of morphic words is given, and several closure results are proved. Closure under morphism and under transduction are the two main results.

Chapter 8, **Frequency of Letters** covers usual frequency and logarithmic frequency of letters in sequences. The Perron–Frobenius theory is developed in detail, with full proofs. It is used to show that frequencies of letters in morphic words, when they exist, are algebraic numbers and they are rational for automatic sequences.

Chapter 9, **Characteristic Words** presents a part of the theory of Sturmian words, concentrating on the number theoretic aspects of characteristic words. This is an excellent choice since it allows a streamlined, not very difficult presentation of the relation to continued fractions expansion, and to Ostrowski number systems.

Chapter 10, **Subwords** is on subword complexity, that is the number of words of length n appearing in an infinite sequence, also the gap between two consecutive apparitions of a finite factor, or the first apparition of a factor in a word. The chapter contains Morse and Hedlund's gap theorem, bound for the subword complexity of morphic words, the characterization of Sturmian words and a theorem of Mignosi relating power-freeness of characteristic sequences to bounded partial quotients in continued fractions. Recurrence, uniform recurrence are also studied.

Chapter 11, **Cobham's Theorem** is devoted to Cobham's theorem on the base dependence of automatic sequences. This is one of the main theorems of the theory, and a detailed and careful proof is given.

Chapter 12, **Formal Power Series** contains the third equivalent definition of automatic sequences, namely by algebraic series over the polynomials over a finite field (Christol's theorem). This chapter studies algebraicity and transcendence of formal power series when the ground field is finite. Using Christol's result, one can reformulate Cobham's theorem and state that certain algebraic series must in fact be rational. Christol's theorem can also be used directly to prove that some series are transcendental.

Chapter 13, **Automatic Real Numbers** is on real numbers associated to infinite sequences, when the symbols in the sequence are considered as digits. Typically, a (automatic) sequence $a_0a_1 \cdots a_n \cdots$ with $0 \leq a_i < b$ is associated with the real number ("automatic" real number) $\sum_{i \geq 0} a_i b^{-i}$. The set of automatic real numbers is shown to be a vector space, but not closed under multiplication. The chapter continues with transcendence results for some automatic real numbers, and for morphic real numbers.

Chapter 14, **Multidimensional Automatic Sequences** extends results on automatic sequences to greater dimensions. It contains again numerous examples, and a discussion of automatic sequences in base $-1+i$. It ends with the Pascal triangle modulo a non-prime number.

Chapter 15, **Automaticity** measures how much a set or a sequence is regular by approximation. More precisely, given a language L, denote by $A_L(n)$ the minimal number of states of finite automata recognizing a language that have the same words as L up to length n. It appears that $A_L(n) \geq (n+3)/2$ for infinitely many n if L is non-regular. Similarly $A_s^k(n)$ is the minimal number of states of finite automata that recognize k-automatic sequences that have the same n initial symbols that s. It is shown that there exist 2-automatic sequences s such that $A_s^k(n) = \Omega(k\sqrt{n})$ for odd $k > 1$.

Chapter 16, **k-Regular Sequences** presents these sequences that are very closely related to rational formal power series in non-commutative variables. Thus, finiteness of automatic sequence transposes to finite dimension for k-regular sequences. The presentation does not rely on the theory of non-commutative power series. A great number of closure properties, and of examples, are given.

Chapter 17, **Physics** is a short chapter on relations between physical concepts and infinite sequences. Two cases are treated, namely the Ising model, especially in relation to the Rudin–Shapiro sequence, and the one-dimensional Schrödinger operator, for which some information about the spectrum is presented.

3 Opinion

This book is the very first book that gathers together results from various fields of mathematics and computer science that appeared in a broad variety of journals. It presents the results in a unified manner, making use of a consistent notation and introduces some good unifying terminology, and including most of the material needed for a comprehensive exposition.

The level of presentation is advanced undergraduate to beginning graduate. Although all proofs are given in full details, with the exception of some explicitly mentioned results, some familiarity with mathematical reasoning is required. Familiarity with basics in formal automata is useful, and a first course in number theory is recommended (by me). There are some places where results from number theory are quoted and used which are not in a first course.

This book is useful also as a handbook for self study by the high number of exercises (460) and as basis for research (85 open problems). It is extremely valuable as a reference text for researches and contains a bibliography of more than 1600 citations to the literature, all commented or placed in context in the text. The stuff is presented with great typographical care. It is extremely difficult to find some mistakes. May be, the fact that the paper [Cassaigne and Karhumäki 1995a] appeared in Europ. J. Combinatorics in 1997.

This book can also be used as a basis for a course on formal languages and number theory at the advanced undergraduate or graduate level. I strongly recommend this excellent book to anybody interested in interaction between theoretical computer science and mathematics.

4 Additional Information Available

There is a website associated to the book at:

http://www.math.uwaterloo.ca/˜shallit/asas.html

(There is a tidle before the 'shallit' .)