The Book Review Column[1]

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In this column we review the following books.

1. **Analytic Combinatorics** by Philippe Flajolet and Robert Sedgewick. Review by Review by Miklós Bóna. Can you really use analytic techniques in combinatorics? Yes! And this book tells you how! More than that- it takes the informal term *Analytic Combinatorics* and defines the term and the field.

2. **Combinatorics The Rota Way** by Joesph P.S. Kung, Gian-Carlo Rota and Catherine H. Yan. Review by John Mount. How did Rota do combinatorics? He tried to refine mere tricks into general techniques. This book shows combinatorics as a field of general methods.

3. **A Course in Enumeration** by Martin Aigner. Review by Peter Boothe. When I think *combinatorics* I think about counting (say) the number of ways to tile a $1 \times n$ with 1×1 and 1×2 tiles. But what if I want to enumerate all the ways to do this? Most questions in counting have an enumeration-analog. This book is, as the title states, a whole course in Enumeration.

4. **A Combinatorial Approach to Matrix Theory And Its Applications** by Richard Brualdi and Dragos Cvetkovic. Review by Miklós Bóna. This books treats combinatorics as a lens, through which we consider the main topics of linear algebra, mainly matrices. It is this combinatorial perspective that makes the book different from other rigorous undergraduate textbooks on linear algebra.

5. **The Annotated Turing** by Charles Petzoid. Review by Kevin A. Wilson. This is Turings original paper with much anotation to make it relevant and readable to the modern reader.

6. **Logicomix** Text by Apostolos Doxiadis and Christos Papadimitriou! Art by Alecos Papadatos and Annie di Donna Review by William Gasarch! This is a comic book! Honest! Its about Bertrand Russell's quest for certainty! Does telling history of math in comic book form work! Read the review to find out!

7. **Proof and Other Dilemmas: Mathematics and Philosophy** Edited by Bonnie Gold & Roger A. Simons. Review by Christopher Pincock. The term *Philosophy of Math* makes most people think of logic and foundations. There are many other issues in the philosophy of math. This book is a set (class? group?) of essays on non-logic aspects of philosophy of math. Included is how much forces outside of math affect math.

8. **Essays in Constructive Mathematics** by Harold M. Edwards. Review by S. C. Coutinho. While the non-constructivists may have won the debate of the early 1900's, there is still much constructivist math being done. Let this book be your guide.

9. **Is Mathematics Inevitable? A Miscellany** Edited by Underwood Dudley. Review by José Guimarães, 2010. This is a book contains essays about mathematics. To quote the review: The book articles can be divided roughly in six categories: 1) comments on Mathematics and mathematicians (four) 2) Mathematics (six) 3) history (ten) 4) teaching (one) 5) humor (one) and simply 6) uninteresting (four).

10. **A Concise Introduction To Languages and Machines** by Alan P. Parkes. Review by Mike Williams. This is an undergraduate automata theory textbook.

11. **A Second Course in Formal Languages and Automata Theory** by Jeffrey Shallit. Review by Kevin A. Wilson. This has many topics that really could be in a first course in Automata theory but usually are not, such as finite-state transducers.

12. **Automata Theory with Modern Applications** by James A. Anderson. Review by Kyriakos N. Sgarbas. This is a standard text in automata theory.

13. **Change is Possible: Stories of Women and Minorities in Mathematics** by Patricia Clark Kenschaft. Review by Sorelle A. Friedler. This book is about the progress that women and minorites *have made!* and *have not made!* in mathematics.

14. Joint review of (1) **Riot at the Calc Exam and other Mathematically Bent Stories** by Colin Adams; (2) **The Great π/e Debate: Which is the Best Number?** by Colin Adams VS Thomas Garrity, Moderated by Edward Burger; (3) **The United States of Mathematics: Presidential Debate** by Colin Adams VS Thomas Garrity, moderated by Edward Burger Review by William Gasarch. The first item is a book, the last two are DVD's. All are works of mathematical fiction. Are they entertaining? Informative? Read the review to find out!

BOOKS I NEED REVIEWED FOR SIGACT NEWS COLUMN

Handbooks

1. *Algorithms and Theory of Computation Handbook* edited by Atallah and Blanton.

Books on Algorithms and Data Structures

1. *Methods in Algorithmic Analysis* by Dobrushkin.

2. *Nonlinear Integer Programming* by Li and Sun.

3. *Binary Quadratic Forms: An Algorithmic Approach* by Buchmann and Vollmer.

4. *Algorithmic Algebraic Combinatorial and Grobner Bases* Edited by Klin, Jones, Jurisic, Muzychuk, Ponomarnko.

5. *Algorithms in Bioinformatics: A Practical Introduction* by Sung.

6. *Introduction to Scheduling* Edited by Robert and Vivien.

7. *Algorithmic Bioprocesses.* Edited by Condon, Harel, Kok, Salomaa, Winfree.

8. *Graph Theory and Interconnection Networks* by Hsu and Lin

Books on Cryptography and Coding Theory

1. *Understanding and Applying Cryptography and Data Security* by Elbirt.
2. *Concurrent Zero-Knowledge* by Alon Rosen.
3. *Secure Key Establishment* by Choo.
4. *Algebraic Cryptanalysis* by Bard
5. *Cryptanalytic Attacks on RSA* by Yan.
6. *Coding for Data and Computer Communications*

Combinatorics and Probability

1. *Combintorial Methods with Computer Applications* by Gross.
2. *Combinatorics: A Guided Tour* by Mazur.
3. *Additive Combinatorics* by Tau and Vu.
4. *Design Theory* by Lindner and Rodger.
5. *Dueling idiots and other probability puzzlers* by Nahin.
6. *Digital Dice: Computational solutions to Practical Probability Problems.* by Nahin.
7. *Elementary Probability for Applications* by Durrett
8. *Polya Urn Modes* by Mahmoud.

Complexity Theory

1. *Logical Foundations of Proof Complexity* by Cook and Nguyen.
2. *Elements of Automata Theory* by Sakarovitch
3. *Handbook of Weighted Automata* by Droste, Kuich, Volger.

Semantics and Programming Languages

1. *The Calculus of Computation: Decision Procedures with Applications to Verification* by Bradley and Manna.
2. *Drawing Programs: The theory and Practice of Schematic Functional Programming* by Addis and Addis.
3. *Semantic techniques in Quantum Computation.* Edited by Gray and Mackie.

4. *From semantics to computer science: Essays in honour of Gilles Kahan* Edited by Bertot, Huet, Levy, Plotkin.

5. *Process Algebra: Equational theories of Communicating Processes* by Baeten, Basten, and Reniers.

Misc

1. *How to solve it* by Polya.

2. *Not always buried deep: A second course in elementary number theory* by Pollack.

3. *Stories about Maxima and Minima* by Tikhomirov.

4. *Difference Equations: From Rabbits to Chaos* by Cull, Flahive, and Robson.

5. *Models of Conflict and Cooperation* by Gillman and Housman.

Review of[2]
Analytic Combinatorics
by Philippe Flajolet and Robert Sedgewick
Published by Cambridge Press, 2009
824 pages, Hardcover
Amazon: $66.00 new, $81.00 used[3]

Review by Miklós Bóna

1 Introduction

This is a very important book, and we will say more about its importance at the end. The goal of the authors is to explain how to use the techniques of real and complex analysis in order to enumerate combinatorial objects.

2 Summary

The book consists of three parts. The first part, which is a necessary prerequisite for the other parts, is about Symbolic Methods. Most combinatorial structures can be put together from simpler parts according to certain rules. For instance, set partitions are just (unordered) sets of blocks, permutations are sets of cycles, compositions are sequences of positive integers, and integer partitions are multisets of positive integers. The authors call structures that can be built up like this *admissible*, and introduce symbolic notation to describe the way in which they are built up. For instance

$$\mathcal{P} = \mathcal{SET}(\mathcal{CYC})$$

[3]This is not a typo. Someone thinks its worth more used then new.

means that the class $\mathcal{P}$ of permutations of an n-element set are nothing else but a set of disjoint cycles on that set so that each element of the set is part of a cycle, while

$$\mathcal{A} = \mathcal{SEQ}(\mathcal{CYC})$$

means that the class $\mathcal{A}$ of alignments is equal to the class of *linearly ordered* set (or sequence) of cycles on the same set.

Then the authors explain how symbolic equations can be turned into equations for the generating functions of the classes that they describe. Chapter I is devoted to unlabeled structures and the ordinary generating functions enumerating their classes, while Chapter II treats labeled structures and their exponential generating functions. The Lagrange Inversion Formula is also covered.

Chapter III takes the discussion to a higher level by introducing bivariate generating functions. It is explained how to use these generating functions to compute the average value of a certain parameter of a combinatorial structure. This technique can be used to compute the average number of cycles in a randomly selected alignment of size n, or the average number of parts in a randomly selected composition of n whose parts have to be in a certain set, and so on. The main idea is that if $A(x,u) = \sum_{n,k} a(n,k)x^n u^k$, where $a(n,k)$ is the number of objects of size n in which the value of a certain parameter is k, then the coefficient of x^n in

$$\frac{d}{du}A(x,u)|_{u=1}$$

is the *cumulative value* of that parameter for all our objects of size n. Hence, dividing that the number by $[x^n]A(x,1)$, that is, by the total number of our objects of size n, we get the average value of the parameter. Taking higher derivatives of the generating function can lead to the computation of higher moments of the parameter.

A spectacular application is that the random composition of the integer n contains about $n/4$ parts equal to 1, about $n/8$ parts equal to 2, and so on, $\frac{n}{2^{i+1}}$ parts equal to i. In particular, the if $i > (\log_2 n) - 1$, then the typical composition of n will not contain a part equal to i. The result describing how the average composition looks like is called the *profile* of the composition. We later find results on the profiles of other objects as well, such as permutations and various kinds of graphs.

The second, and most extensive part of the book is on Asymptotic Computation. This is the subject of Chapters IV through VIII. Some knowledge of complex analysis is useful for this chapters, but it is not necessary; most of the needed techniques can be learned from this book as well. First, we are explained the notion of analytic functions and their singularities, with the fundamental theorem that the exponential order of the coefficients of an analytic function f is equal to $1/\rho$, where ρ is the absolute value of the singularity ρ of f that is closest to 0. For rational functions, which are the ratio of two polynomials, and for meromorphic functions, which are the ratio of two analytic functions, more precise results are given.

We then move on to *Singularity Analysis*, which is perhaps the most unique part of the book. It is explained that when looking for the growth of the coefficients of a power series, it is not simply the location of the singularities that matters, but also their *number* (there can be several singularities on the same circle around 0), and their *type*. For instance, while $f(x) = \frac{1}{1-x}$, $g(x) = 1/\sqrt{1-x}$ and $h(x) = \log(1/(1-x))$ all have a unique singularity at $x = 1$, the growth rates of their coefficients are different in their subexponential terms. In particular, $f_n = 1$, $g_n \sim \frac{1}{\sqrt{\pi n}}$, and $h_n = 1/n$. The authors show that while the location of the dominant singularity determines the exponential order of the coefficients, the type and number of these singularities determines the subexponential factors. The three examples above are representative to the three types of singularities treated here, namely poles, square-root type, and logarithmic type singularities. We again find some spectacular and surprising applications besides the standard ones.

A large family of applications concerns combinatorial classes that are enumerated by a generating function $T(x)$ that satisfies an equation of the type $T(x) = x\phi(T(x))$, for some non-linear function ϕ. Classes of rooted trees for which this holds are called *simple tree varieties*. Several sections of Chapter VII are devoted to exploring the parameters of these varieties. For instance, if τ is the unique positive real root of the equation $\phi(x) = x\phi'(x)$, then it is shown that the radius of convergence $T(x)$ is $\rho = \tau/\phi(\tau))$. If n, the number of vertices of the trees, goes to infinity, then the probability generating function of the degree of the root is $u\phi'(\tau u)/\phi'(\tau)$. For example, if our tree variety is that of unary-binary unlabeled plane trees, (each non-leaf vertex has one or two children), then $\phi(w) = 1 + w + w^2$, so $\tau = 1$, and $\rho = 1/3$. The mentioned probability generating function is $u/3 + 2u^2/3$, meaning that for large n, in about two-third of all unary-binary trees on n vertices, the root will have two children. Several other parameters, such as the average number of vertices at distance k from the root, the average number of vertices of degree k, and the average path length can easily be computed once ϕ and τ are known.

Chapter VIII takes the discussion one level higher by covering the *Saddle Point Method*. This is the only part of the book where familiarity with complex integrals, at least to the level of knowing Cauchy's coefficient formula,

$$[z^n]G(z) = \frac{1}{2i\pi} \int_C G(z) \frac{dz}{z^{n+1}}$$

is necessary to understand the essence of the method (if not, strictly speaking, to be able to use the method in simple cases). Here C is a contour that encircles the origin, lies within the domain where G is analytic, and is positively oriented.

"Saddle points" are points where the derivative of a function is zero, while the function itself is not. The saddle point method is to be applied when singularity analysis fails, either because the function at hand has no singularities, such as $e^{x+x^2/2}$ (the exponential generating function of involutions), or when the function has singularities not covered by the other methods of singularity analysis, such as $e^{1/(1-x)}$ (the exponential generating function of broken permutations. The analytic basis of the method is selecting a contour C that goes through or very near a saddle point of G, and then applying Cauchy's coefficient formula, displayed above. Besides the two mentioned examples, the method is applied to set partitions, various classes of permutations, and many other admissible structures (structures that can be built up by composing sets, lines, and cycles).

Finally, Chapter IX, which constitutes the third, shortest part of the book, is on the probabilistic aspects of analytic combinatorics. Given a combinatorial class and a parameter, such as permutations and their number of cycles, one can ask what the distribution of the parameter is when the objects are of size n. Then one can ask what happens to that distribution as n goes to infinity. Various notions of convergence are discussed for these distributions. Then the authors show how to use multivariate generating functions to prove results about the limits of sequences of distributions. In the mentioned example, as n goes to infinity, the distribution of the number of cycles of permutations of size n converges to a normal distribution.

The book concludes with three appendices, on Elementary Combinatorics, Complex Analysis, and Probability, which make the book self-contained.

3 Opinion

This reviewer has been teaching a graduate class from this book this semester. It is a serious time commitment. The book has enough material for three or more semesters if all details are covered, so the instructor faces very serious choices as to which topics to include. As the discussion advances, the part of analysis grows at the expense of combinatorics, so it is the instructor's task to keep balance. Students in the class like the class and the book in general, though they disliked the fact that Exercises are not clearly marked in the book (there are many examples where filling in the details is left as an exercise, but the difficulty level of these was often not clear for the students at the outset).

However, these are minor points of criticism, and most books that are meant both for students and researchers will have a few minor problems like that. More importantly, the book does nothing less than it *creates* the notion of Analytic Combinatorics. Before this book, when someone said "Analytic Combinatorics", it was not clear what he meant by it. There was no wide consensus on what works belong there, unlike in Algebraic Combinatorics, or Probabilistic Combinatorics. The Online Journal of Analytic Combinatorics was only started a few years ago. Because of the breadth, and depth of topical coverage, the highly applicable results and the enjoyable writing that characterize this book, Analytic Combinatorics is now defined. The authors wrote the book on it.

Review of[4]
Combinatorics The Rota Way
by by Joesph P.S. Kung, Gian-Carlo Rota and Catherine H. Yan
Published by Cambridge Press, 2009
396 pages, Softcover
Amazon: $32.00

Review by John Mount `jmount@win-vector.com`

1 Introduction

Combinatorics, as it matures, becomes harder to succinctly describe. The field has progressed from the basic study of finite sets and counting techniques to being the discipline where questions involving counting, graphs, connectivity, mappings and partial orders all naturally reside. But the objects that combinatorics studies turn out not to be the correct foundation to support modern combinatorial methods. Many combinatorial methods were dismissed as mere technique until combinatorics expanded to include the natural domains of these methods: lattices, formal power series, valuation rings, matroids and many diverse algebras. One person who pushed hard for this coherence and unity was Gian-Carlo Rota.

An example of a high-school level combinatorial trick is proving the equation

$$\sum_{i=0}^{n} \binom{n}{i} = 2^n$$

by applying the binomial theorem to $(1+1)^n$. This trick is transformed into a method when you recognize that you really should be working in the ring of formal power series and invent the Umbral

Calculus. With the Umbral Calculus you can use the equivalence of the following two equations:

$$\begin{aligned} b^n &= (a+1)^n = \sum_{i=0}^{n} \binom{n}{i} a^i \\ a^n &= (b-1)^n = \sum_{i=0}^{n} (-1)^{n-i} \binom{n}{i} b^i \end{aligned}$$

(i.e. $b = a+1$ is equivalent to $a = b-1$) to prove that for any two arbitrary infinite sequences a_i, b_i the following two statements are also equivalent:

$$b_n = \sum_{i=0}^{n} \binom{n}{i} a_i \text{ for all } n \tag{1}$$

$$a_n = \sum_{i=0}^{n} (-1)^{n-i} \binom{n}{i} b_i \text{ for all } n. \tag{2}$$

For example: we could pick $a_i = i$ and substitute it into Equation 1. With some work we see this implies $b_i = 2^{i-1} i$.[5] Then by the Umbral result we know Equation 2 must also be true so we get a new identity: $n = \sum_{i=0}^{n} (-1)^{n-i} \binom{n}{i} 2^{i-1} i$. This algebraic production of a new identity is very different than the classical method of "counting two ways" (or being lucky enough to come up with a clever bijection to prove the identity).

2 Summary

The book "Combinatorics the Rota Way" is itself hard to succinctly describe. The first and third authors tell of writing this book using notes from the Massachusetts Institute of Technology's course 18.315 collected over a span of more than 30 years. Gian-Carlo Rota himself was added as a posthumous author. The book itself contains more than a single course-year's worth of material and is packed very densely.

The book's emphasis is abstract and algebraic. The exercises are not to teach, but are instead to identify applications of combinatorics in other mathematical disciplines. The book is the product of a strong push to demonstrate many combinatorial methods in their most powerful, but not most obvious, forms. This work is clearly a labor of love and contains some remarkable material. However, due to the large breadth of the work not much time is spent on motivation or on concrete examples.

2.1 Chapter 1: Sets, Functions and Relations

The first chapter covers the definitional foundations of combinatorics: sets, lattices, partial orders, functions and relations. These are the discrete objects that the book will reason about by later building more complicated algebraic objects. This section is very dense and reads like a compressed Bourbaki treatment of discrete mathematics.

[5] For this use the binomial theorem to expand $(1+x)^n$, differentiate with respect to x and then substitute in $x = 1$.

One portion of this chapter that is problematic is the section on entropy that seems to serve no purpose other than to prepare the reader for exercise 1.4.10 which demonstrates an abstraction of entropy. Also, exercises 1.2.5(j,k) are needlessly cruel in asking the reader to recreate the Robertson-Seymour graph minor theorem. There have been books where the reader is successfully guided through a major result by exercises, such as the Weak Perfect Graph Theorem in Lovász's "Combinatorial Problems and Exercises", but this book is not structured in that manner.

2.2 Chapter 2: Matching Theory

The second chapter is a welcome change in tone and opens with a quote from Harper and Rota describing matching theory and a clever 1979 Putnam exam problem is worked into the exercises and solutions. Central to the chapter is "marriage theorem", which determines when matchings are possible. Also discussed is Birkhoff's Theorem, which states that every doubly stochastic matrix is a convex combination of permutations matrices, which relates matchings to matrices. The text is lively and includes a number of well-researched asides, such as the origin of the name "The Hungarian Method." However, there are some problems with forward reference: for example the reader is asked to work a couple of exercise (2.4.5 and 2.4.6) using the Binet-Cauchy formula, which isn't discussed at length until chapter 6.

2.3 Chapter 3: Partially Ordered Sets and Lattices

This chapter begins with a very exciting presentation of the Möbius Function (the convolutional inverse of what is essentially the indicator function of a partial order). It is a real pleasure to see this material well presented in a general lattice setting, instead of the more common and specialized number theoretic setting. The chapter moves on to chains (ordered sequences in lattices) and anti-chains (sets of incomparable elements) in partial orders. The authors present Dilworth's theorem,[6] which shows that every large partial order must have at least one large chain or one large anti-chain. From this we move on to Sperner Theory, which relates counting anti-chains to binomial coefficients. Chapter 3 concludes with valuation rings and Möbius Algebras: a transition to the more algebraic style found in Chapter 4.

2.4 Chapter 4: Generating Functions and the Umbral Calculus

This is a key chapter. The book introduces the Umbral Calculus, a transform space automating the manipulation of generating functions. The algebra of delta operators is introduced, which provides an abstraction of differentiation. Finally co-algebras are explored, which abstract the processes of factoring.

A rare (and unfortunate) typo on page-190 mis-defines a basic sequence $p_n(x)$ for the delta operator Q as obeying $Qp_n(x) = p_{n-1}(x)$ instead of the correct equation: $Qp_n(x) = np_{n-1}(x)$. A careful reader can spot the mistake as it is inconsistent with the the subsequent demonstrations and uses.

[6]This is just about the only Ramsey-theoretic style result in the book.

2.5 Chapter 5: Symmetric Functions and Baxter Algebras

This chapter treats a number of important algebraic topics. Symmetric functions are studied and identified as being the obvious class of functions that contains all of the well know generating functions already studied. Pólya's Enumeration Theory, which is the method of counting the number of equivalence classes of distinct arrangements, is given a very interesting exposition. But the book skips the classic examples and exercises, such as counting the number of ways to construct distinct necklaces from colored beads, that are needed for the topic to be fully approachable. Baxter Algebras, which abstract both summation and integration by parts, are introduced and via a study the sequence shift operator. By this point the book has abstract versions of both differentiation and integration, providing a combinatorial groundwork to prove theorems on "the calculus" that are more general than is possible in any one theory of differentiation or integration.

2.6 Chapter 6: Determinants, Matrices and Polynomials

This chapter is most similar to classical polynomial invariant theory, the study of symmetric functions of the roots of polynomials such as the discriminant. A major theme of this chapter is the study of the relations between properties of polynomial coefficients and the locations of roots of the polynomials. The study of matrices brings us to the remarkable Binet-Cauchy Formula for the determinant of a product of matrices. The results are deep, but it is a shame that more time isn't spent on simple concrete applications such as using the Binet-Cauchy formula to count the number of spanning trees in a graph. This chapter reveals the parts of combinatorics that come from analysis and the study of locations of roots of polynomials (via group theory), in contrast to the parts that come from enumerating finite sets, linear algebra and abstract algebra. This is also the chapter where the exterior algebra, a favorite tool of Rota's, is most discussed.

A typo on page 275 (a potentially confusing comma in the definition of the $eval()$ operation) can be recovered from because the authors have the nice habit of explicitly calling out the domain and range of functions.

3 Opinion

Some important questions about this book are: is Gian-Carlo Rota a coauthor, what is the purpose of the book and who is the best audience?

Gian-Carlo Rota seems appropriately labeled as a co-author, as clearly a lot of his work went into the book. The book is not suitable to be used as an introductory text book or as a reference. It is a book meant to be read. The ideal audience is capable of graduate level mathematics, is comfortable with a high degree of abstraction and algebra and is already familiar with many of the structures and techniques of combinatorics: sets, graphs, matrices, alternating sequences and generating functions. A mathematician or computer scientist wanting to learn more about the science of combinatorics will find a good read here.

The book works best as a second read of the topics covered. If you already know of a combinatorial method, like Pólya's Enumeration Theory, this book is a good place to find the starting point for an alternate and powerful treatment of the topic. The book admits to not being self contained, and has a few forward-reference problems. However, this is forgivable when you realize the goal of this book is not to teach some easy discrete mathematics before you move on to analysis, but to extract the important combinatorial methods and themes from all of mathematics.

The content is well written, very accurate and well edited. The index is good, but not quite up to the job. The bibliography is very good and divided into three useful sections: papers by Gian-Carlo Rota and coworkers, books for further reading and a section of references.

We close with a extract from the book at hand. Many mathematicians have used the phrase "merely combinatorial proof" as a phrase of dismissal. However, when properly founded, combinatorial proofs are in fact more general than proofs that depend on additional specific details from the original problem domain. The authors take some justifiable pleasure in including points like: "Hilbert's basis theorem is equivalent to the 'trivial combinatorial fact' given in Gordan's lemma." This is certainly a taste of combinatorics the Rota way.

Review[7] of
A Course in Enumeration
by Martin Aigner
Springer, 2007
555 pages, Hardcover, New $65.00,Springer 2007

Review by Peter Boothe
Peter Boothe `peter.boothe@manhattan.edu`

1 Introduction

Combinatorial enumeration is the study of counting the numbers of members in finite sets in order to determine the size of said sets. A classic example of combinatorial enumeration is the question of how many ways there are to tile a board of size 2-by-n using 1-by-2 dominoes. The equally classic solution is that the number of tilings $t(n)$ of a board of length n is exactly $t(n) = t(n-1) + t(n-2)$, that there is exactly one way of tiling a board of length 1, there two ways of tiling a board of length 2, and therefore, $t(n) = F_{n+1}$ for all positive n. There must also be some careful definition of what equality means for members of a set, as our tiling results would be wrong if we could distinguish between the dominoes laid down on the board.

Enumeration has a rich connection with computer science — in order to build a "generate-and-test" algorithm, one must be able to generate all the possible candidate solutions. Furthermore, in order to generate all candidate solutions, one must at the very least be able to count them. In this sense, enumeration can be seen as the precursor to generation. A word to the interested computer scientist is in order: enumeration means that each element of the set is counted, it does not mean that each element of the set is generated. This book does not provide computer science algorithms for set element generation, it provides mathematical analysis of combinatorial sets, as is appropriate for a book in the Springer series *Graduate Texts in Mathematics*.

2 Summary

The book is divided into three parts: Basics, Methods, and Topics, and each part consists of multiple chapters. Each section of each chapter ends with exercises, many with solutions in the back of the book. Each chapter ends with a "Highlight" result, chosen for its combination of brevity and beauty, which illustrates an advanced result using the techniques in the chapter.

Basics contains two chapters: Fundamental Coefficients (which includes material on Stirling numbers, Gaussian coefficients, set partitions, etc.) and Formal Series and Infinite Matrices. The "Highlight" section of the second chapter is a particularly nice discussion of random walks on the the integer lattices $\mathbb{Z}^1, \mathbb{Z}^2, \mathbb{Z}^3, \ldots$ ending with a proof that the probability a random walk eventually returns to the origin is 1 for $\mathbb{Z}^1$ and $\mathbb{Z}^2$, but is less than 1 for $\mathbb{Z}^d$ where $d \geq 3$. The material in both chapters is developed from base principles, but prior exposure to the ideas and concepts helps the brevity of development not seem too terse.

Methods is where the book's explanations begin to assume that the material may be all or wholly new to the reader. The methods section contains four chapters: Generating Functions, Hypergeometric Summation, Sieve Methods, and Enumeration of Patterns. The chapter of each title is an accurate guide to that chapter's contents.

The last section, **Topics**, consists of material which is less general than the previous section and more specialized to specific sub-areas of combinatorics and enumeration. The chapters are: The Catalan Connection, a wide-ranging discussion of applications of the Catalan numbers and related constructions; Symmetric Functions; Counting Polynomials, which contains a superb graph polynomial exposition as well as an introduction to knot polynomials; and Models from Statistical Physics.

3 Opinion

Martin Aigner has a reputation as a good expositor of mathematics (I showed to book to a mathematician colleague and she told me I was in good hands), and the book does not disappoint. The explanations, while often brief, are quite good. The only caution I would give is that the material in the first part of the book (Basics) also is the material that is explained the most briefly. It seems the Basics part of the book is meant to be a combination of review and filling in of cracks rather than an exposition of completely new material. The reader looking for an introduction to e.g. Sterling numbers or binomial coefficients or probability generating functions should probably look for a more introductory text.

As the book gets more and more advanced, the explanations grow correspondingly in size. In the advanced section, (Topics) the book contains the clearest explanation of graph polynomials that I have ever found. He develops the chromatic polynomial, flow polynomial, and Tutte polynomial in extremely readable fashion. His prose ended up clarifying material that I had been trying to get an intuitive grasp on for quite a while. I was also grateful that the author of the book put the answers to selected exercises in the back of the book. Combinatorics can be quite subtle, so having a reference answer with which to check my own work was of great value. Professors interested in teaching out of the book, students taking classes from those professors, and people independently studying the subject should all be made aware (for entirely different reasons) that exercises with $\triangleright$ next to them have solutions in the back of the book.

In summary, the book is not easy math and is not for the faint of heart or mathematically inexperienced, or those looking for generation algorithms. With a bit of background, however, the book contains good and readable expositions of an interesting and beautiful subject.

Review of[8] of
A Combinatorial Approach to Matrix Theory And Its Applications
by Richard Brualdi and Dragos Cvetkovic
Published by Cambridge Press, 2009
824 pages, Hardcover
Amazon: $76.00

1 Introduction

The area on the borderline of Combinatorics and Linear Algebra is underserved by textbooks. There are at least two possible approaches to this area. One would be to treat Combinatorics as the goal and Linear Algebra as the tool. In that approach, the authors would discuss applications of Linear Algebra to Combinatorics, that is, proving combinatorial theorems by first translating them to the language of Linear Algebra, and then proving them in that setup, using facts from Linear Algebra. The other approach, chosen by the authors of this book, treats combinatorics as a lens, through which we consider the main topics of our study, matrices. It is this combinatorial perspective that makes the book different from other rigorous undergraduate textbooks on linear algebra. This is why this review will highlight precisely those points in which the book, using its combinatorial lense, brings something new into the discussion of the otherwise well-known topics.

2 Summary

Chapter 1 is a very basic introduction to both combinatorics and linear algebra. The authors introduce the concept of graphs, trees, matchings, vertex covers, as well as fields and vector spaces. The most advanced theorem is that of Konig stating that in any bipartite graph, the size of the minimal vertex cover (the smallest set of vertices that intersects every edge) is the same as the size of the largest matching (the largest set of pairwise vertex-disjoint edges).

Chapter 2 is about basic matrix operations. In addition to what is expected, that is, sum and product of matrices, properties of partitioned matrices, there is an other definition here that this reviewer found interesting. That is the Konig digraph of a matrix, which is obtained as follows. The Konig digraph of an $m \times n$ matrix A is a directed graph on $m + n$ vertices. Its underlying undirected graph is a complete bipartite graph with m vertices in color class R and n vertices in color class C. All edges point from R to C. Each edge has a weight, and the weight from R_i to C_j is equal to $A_{i,j}$.

This reviewer is a combinatorialist, and as such, is used to matrices being created from combinatorial objects. It is refreshing to see the other direction– in this example, the matrix is given, and we create a combinatorial object– a graph– from that. Permutation matrices are also introduced in this chapter, enabling the teacher to make a point about the ubiquity of matrix groups.

Chapter 3 is about powers of matrices and their combinatorial meaning. To a square matrix A of size $n \times n$, the authors associate the directed graph $D(A)$ on n vertices in which the edge from vertex i to vertex j has weight $A_{i,j}$. So unlike the Konig digraph of a matrix, $D(A)$ is not necessarily bipartite. The central theorem of the chapter is that if the weight of a walk in $D(A)$ is the product of the weight of the edges in that walk, then the sum of the weights of all walks from vertex i to vertex j in k steps is the (i, j) entry of the matrix A^k. Note that in the special case when $A_{i,j}$ is the number of edges from i to j, this means that $A^k_{i,j}$ is the *number* of length-k walks from i to j.

An interesting application is that A is nilpotent if $D(A)$ has no cycles. If A is non-negative, then the reverse implication also holds.

Chapter 4 is about determinants. There is one difference from the usual treatment of the topic. The first definition that the authors give for the determinant of the $n \times n$ matrix A uses graph. Let $D^*(A) = D(A^T)$, where $D(B)$ is as was defined in the preceding chapter. A *linear subdigraph* of a digraph is a subdigraph in which every vertex has indegree and outdegree 1. Then the authors set

$$\det A = (-1)^n \sum_{L \in L(A)} (-1)^{c(L)} w(L),$$

where the summation is over all linear subdigraphs L of $D^*(A)$, and $c(L)$ is the number of cycles contained in L. As always, $w(L)$ is the weight of L, that is, the product of the weights of the edges of L.

The authors prove the well-known properties of the determinant using this definition. Then they prove that this definition of the determinant agrees with the classic one.

Chapter 5 is a very short chapter on the inverse of a matrix. What makes this chapter different from the corresponding chapters in other linear algebra textbook is that the authors use the notions of Coates digraph, linear subdigraphs, and $c(L)$ to provide a formula for the entries of the inverse of a matrix. The formula is complicated, but it is complicated using traditional definitions, too.

Chapter 6 is a longer chapter on one of the most important applications of matrices, namely solving systems of linear equations. We learn the classic Cramer rule, and versions of it in which the numerator is computed from digraphs of matrices. While this is conceptually cumbersome (why create a matrix from the graph and then compute some numbers from it instead of using the matrix directly), it is computationally often easier than the alternative, especially when the matrices at hand are *sparse*, that is, when they have only a few nonzero elements.

Next comes an interesting chapter on eigenvalues, eigenvectors, and Jordan matrices. The most interesting part is a digraph-based proof of the classic Cayley-Hamilton theorem. That theorem says that every square matrix is a root of its own characteristic polynomial. This is a statement of the form $\sum_{k=0}^{n} c_k A^k = 0$, where A is any square matrix, and the c_k are some computable coefficients, every other one of which is negative. Therefore, we can prove the theorem if we can show that in every position, the summands with a positive sign and the summands with a negative sign cancel each other. The authors interpret all these summands combinatorially (as paths in digraphs), and then they pair them up.

This reviewer found Chapters 8 and 9 the most interesting ones in the book, perhaps because he did not see these kind of chapters in other books on matrices. Chapter 8 is about matrices with non-negative (and sometimes, positive) entries. A square matrix with non-negative entries is called irreducible if its digraph $D(A)$ is strongly connected. It is proved that this occurs if and only if $(I + A)^{n-1}$ has only positive entries. The authors then discuss the Perron-Frobenius theorem. In their treatment, the theorem has six parts, and takes an entire page to state. The most important claim is that the largest positive eigenvalue of the non-negative matrix A is at least as large as the absolute value of any other eigenvalue. This eigenvalue is sometimes called the Perron eigenvalue. The last section of the chapter will be of interest to graph theorists since it is about the spectra (set of eigenvalues) of matrices that are defined by graphs, such as adjacency matrices. For such matrices, we are given a combinatorial interpretation of the coefficients of the characteristic polynomial. We are also given a set of equations satisfied by the spectra of such matrices.

Chapter 9 is about a variety of other topics, typically not found in other textbooks. These start by the tensor product and Kronecker product of matrices. Then comes an interesting section explaining the locations of the eigenvalues of a matrix in the complex plane. The authors prove that for all eigenvalue λ of A, there exists an index i so that

$$|\lambda - a_{ii}| < \sum_{j \neq i} |a_{i,j}|.$$

An easy corollary of this is that if A is *diagonally dominant*, that is, if its diagonal entries are larger than the sum of the absolute values of the non-diagonal entries in their row, then A is invertible, since 0 is not an eigenvalue of A. Finally, we hear about permanents, and their combinatorial interpretations.

Chapter 10 is about selected applications. There is one section with showing an application to electrical engineering, one showing an application to physics, and one showing an application to chemistry. Matrices are just about everywhere in science, so any such selection will be highly arbitrary. The authors say that this chapter is for those readers who are not mathematicians, but use mathematics as a tool. This reviewer is somewhat sceptical of this; he thinks that those readers would not read this book at all, and, in their own field, they would want many more examples of applications than this selection. Perhaps it is the mathematics students who read the book so far who will benefit from the chapter in that they will see that the material they have just learned is very applicable.

3 Opinion

There are between 10 and 15 exercises at the end of each chapter. The reviewer believes that combinatorialists will use the book as a reference material when they need to understand a particular point of interplay between matrix algebra and combinatorics. The book is also a good choice for a reading course for an interested advanced undergraduate student.

Review of[9]
The Annotated Turing
Author: Charles Petzoid
Publisher: Wiley, 2008
ISBN: 978-0-470-22905-7, $29.99

Reviewer: Kevin A. Wilson (kwilson@rti.org)

1 Overview

The Turing machine is a theoretical model of computation that is typically studied as part of a theoretical computer science class at the undergraduate level. Such a course typically introduces fundamental theoretical computer science concepts including several formal models of computation and their associated languages. Most often included are finite state automata and regular languages, push-down automata and context-free grammars. Such classes typically culminate with an introduction to Turing machines. While a significant amount of time is spent on the definition of a Turing machine, and a discussion of its computational power, most students rarely take the time to read Alan Turing's original 1936 paper that, in many ways, provides the theoretical underpinnings for modern Computer Science. Perhaps this can be attributed to the paper's complexity, questionable organization, and confusing terminology. While the paper is central to modern computer science, more modern, succinct treatments, such as those found in standard textbooks provide a significantly gentler introduction to the area.

Petzold's goal in this book is to bring the original Turing paper to the (computer science) masses by providing detailed annotation alongside the original paper to explain, inform, and clarify Turing's pivotal ideas. While clearly focused on the Turing machine, the book also provides a significant amount of context including discussion of the history of mathematics beginning with Diophantus, David Hilbert's famous 1900 address, Turing's life and education, and various mathematical foundations.

2 Review

The book is split into four discrete sections, each containing multiple chapters. The first section of the book covers the necessary mathematical foundations and provides a historical perspective for Turing's work (chapters 1-3). The second section discusses "computable numbers" and forms the major part of the book (chapters 4-11). This section discusses the nuts and bolts of Turing's computing machines. The third section describes "Das Entscheidungsproblem[10]" covering logic and computability, the notion of computable functions, and Turing's major proof (chapters 12-16). Sections two and three together provide a complete analysis of Turing's paper. Finally, section four discusses the future implications of Turing's work (chapters 17-18). Each of these sections and their individual chapters are described in detail below.

[10] "Das Entscheidungsproblem" is the problem of creating an algorithm to determine whether, given a description of a formal computing language, and a statement of that language, it is possible to decide conclusively if the statement is true or false.

Chapter 1 - This Tomb Holds Diophantus. Chapter 1 begins with an overview of ancient mathematics beginning with Diophantus and his famous *Arithmetica*, the mathematical tome that provided the beginnings of what we now refer to as algebra. Diophantus, in the third century of the Common Era, studied the question of whether, given an equation, does it have a solution in integers? This leads to the more general decision problem: given an equation, is it possible to determine if it has an integer solution? If this problem was decidable then other problems such as Fermat's last theorem may be able to be solved in the same way (although complexity issues might make this approach infeasible). We will return to this problem in Chapter 18.

Chapter 2 - The Irrational and Transcendental. This chapter provides some numerical background in preparation for some of the later discussion on countability. Rational numbers are discussed in the context of the algebraic equations defined in chapter 1. The concept that there exist real numbers that do not form solutions to such equations was discovered by Euler and these are referred to as Transcendental numbers. The existence of these numbers is of critical importance to Turing's work as it led to several foundations, particularly the definition of cardinality and the seminal, if controversial, work by Cantor.

Chapter 3 - Centuries of Progress. Chapter 3 describes the centuries of progress, beginning in the 18th century, that culminated in David Hilbert's famous turn-of-the-century address in Paris. In this address, Hilbert posed a series of problems that he expected to be solved in the next century. One such problem was the "Entscheidung" - to determine the solvability of a Diophantine equation. The key notion presented in this chapter is that Hilbert was looking for a *decision procedure* or *algorithm* for determining whether a given equation was solvable or not.

Chapter 4 - The Education of Alan Turing. With mathematical preliminaries covered, the author shifts focus onto the young Alan Turing, describing in detail Turing's educational background and how he came to write his seminal paper "On Computable Numbers, with an Application to the Entscheidungsproblem." This chapter starts the dissection, presentation, and analysis of Turing's paper, covering the preamble and basic definitions. The most important aspect of this chapter is the brief discussion of circular and circle-free machines. We now refer to Turing's circle-free machines as halting machines.

Chapter 5 - Machines at Work. This chapter provides a detailed introduction to the mechanics of what we now know as the Turning machine. The machines are described using Turing's original notation, with additional supporting text used to explain the many inconsistencies. The basic operation of the machine is discussed and the major parts described. Techniques such as reading from and writing to the tape, definition of configurations, and use of configuration tables are described in great detail. Specific examples include the calculation of rational numbers.

Chapter 6 - Addition and Multiplication. Chapter 6 continues with some more complex applications of Turing machines, including addition, multiplication, and calculation of a square root. Each of these operations is described in detail and in terms of the various configurations and moves of the Turing machine. Different steps of the operations are grouped together and labeled in order to illustrate the logical structure of these basic "computer programs". This collection of configuration tables can be reused, as is discussed in the next chapter. This chapter also describes Turing's initial contact with Alonzo Church, a partnership that would prove to be most productive.

Chapter 7 - Also Known as Subroutines. The natural extension of the configuration tables discussed in the previous chapter is the labeling of these tables and subsequent use of these labels in program flow. Chapter 7 discusses this scenario, presenting a shorthand mechanism for encapsulating groups of configurations that exhibit certain repeatable functionality. By using an "abbreviated table" references to external functions can be used to refer to predefined functionality. This represents the first use of the word function in the computing literature. Also in this chapter, the concepts of variables and supplied arguments to functions are introduced.

Chapter 8 - Everything is a Number. Chapter 8 provides the theoretical underpinnings for the Universal Turing Machine. In this chapter, the author discusses the notion of a standard description of a machine, reduced to a numeric representation known as a "description number". Here Turing's work at Bletchley Park and the decoding of the Enigma machine provides some historical insight into these developments. A concrete example of the Turing machine that calculates 1/3 is provided and the notion that any computer program can be represented numerically is discussed, with Microsoft Word being used as an example. Also in this chapter, Turing's description of circle-free machines comes to fruition with the assertion that there is no algorithmic method for determining whether a given machine is circle-free or not.

Chapter 9 - The Universal Machine. This chapter builds on the previous by defining the Universal Turing Machine - a machine that can take an input and simulate another machine when supplied with a standard description. Here Turing, and the author, provide a detailed exposition of the inner workings of this universal machine at the individual cell level. This level of detail is critical to the rest of the paper and so is explored and explained in great detail. It is here that the author identifies several mistakes in Turing's original paper (corrections that were subsequently made by Turing on the advice of Emil Post and Donald Davis). The creation of the Universal Turing Machine that can be "programmed" to carry out the operation of any computing machine forces the question: "Did Turing invent the computer?"

Chapter 10 - Computers and Computability. The transition from the theoretical to the practical is discussed in this chapter, including Turing's partnership with Von Neumann, development of the EDVAC computer, and development of Turing's Automatic Computing Engine (ACE), the first general purpose computer. Prior to its invention, computers had been envisioned as specialized devices built to perform a specific task, rather than general purpose "programmable" machines. The ACE was also the first incarnation of the first binary-based machine, justified by Turing's recognition that "it is so easy to produce mechanisms which have two positions of stability." The chapter concludes with discussion of Turing's 1951 "programmers manual" for the new Mark I computer, and the application of Cantor's diagonal argument to show that there is no algorithm to identify whether a given machine is "circle-free". This represents the first formulation of what we now know as the Halting problem.

Chapter 11 - Of Machines and Men. The focus of Chapter 11 is the Turing's philosophical comparison of the human "computer" with his computing machine. In essence, a human performing a given task is reduced to a representation such that the task can be modeled on a finite-state device using an infinite tape. In preparation for the Entscheidungsproblem, Hilbert functional calculus is also discussed. From a historical perspective, Turing's view of artificial intelligence and the now-famous "Turing test" are discussed. The chapter concludes with biographical details of Turing's suicide at the age of 41, which occurred in 1954, through the ingestion of cyanide.

Chapter 12 - Logic and Computability. Chapter 12 presents a 20-page introduction to mathematical logic in preparation for the discussion of the remainder of Turing's paper. This chapter begins a discussion of Hilbert functional calculus involving a finite number of symbols and defines a computing machine in terms of first-order logic, paving the way for the next chapter on computable functions.

Chapter 13 - Computable Functions. Chapter 13 describes, in mathematical terms, the concept of computable functions, including dependent and independent variables, and domain and range. The computation of a series of functions using Turing machines is also discussed in detail. A major focus of the chapter is the introduction and proof of several theorems of computability. Dedekind's theorem for real numbers is described along with a similar version valid for computable numbers. This chapter completes all the prerequisites for the proof that the Entscheidungsproblem has no solution.

Chapter 14 - The Major Proof. This chapter represents the culmination of the paper and uses the structure Turing has constructed up to this point to prove that the Entscheidungsproblem for first-order logic has no solution. In this chapter Turing refers to the Entscheidungsproblem as general method, or *decision procedure*. Proof is based on contradiction. He first constructs a theoretical Turing machine capable to solving the problem and then proves that it is not possible to construct such a machine, thus generating a contradiction and proving that the problem is insolvable.

Chapter 15 - The Lambda Calculus. Prior to publishing his paper, Turing became aware of Alonzo Church's Lambda Calculus. Church had used Lambda calculus to prove that there is no general decision procedure for first-order predicate logic. In essence this proof was equivalent to Turing's proof. Prior to publication Turing added an appendix to his paper to elucidate and prove this equivalence.

Chapter 16 - Conceiving the Continuum. Chapter 16 describes some of the controversies and disagreements that took place in history of mathematics, particularly relating to the concept of infinity. Different mathematical/philosophical groups are described including realists (such as Gödel) and the constructivists. Much of the chapter focuses on the controversy surrounding Cantor's work on infinity and the continuum.

Chapter 17 - Is Everything a Turing Machine? This chapter discusses the impact of Turing's paper, how Turing machines relate to other, completing models, and the connection between Turing machines and the human mind. The chapter also articulates the development of automata theory, and briefly discusses the work of Kleene and Shannon. Three equivalent notions are considered: the Turing machine, Gödel's recursive functions, and Church's λ-functions. The chapter ends with a brief discussion of whether the human brain is a classical computer, or a quantum computer.

Chapter 18 - The Long Sleep of Diophantus. Although Turing and Church proved that there could be no general decision procedure for first-order logic, Hilbert's original tenth problem [11] had yet to be proven. It was eventually proven in 1970 by Julia Robinson, a mathematician from Arizona, and Yuri Matiyaysevich, a 22-year-old Russian graduate student.

3 Opinion

This book is a well-researched and extremely thorough analysis of Turing's seminal paper and as such provides a detailed commentary of the paper, clearly explaining the often confusing terminology and notation used by Turing. The author provides substantial theoretical and historical background that allows the reader to understand how Turing's work fits within the overall mathematical

[11] Determination of the solvability of a Diophantine equation. Given a polynomial equation with any number of unknown quantities and with integer coefficients: To devise a process according to which it can be determined by a finite number of operations whether the equation is solvable in integers.

universe. In this respect, the book is a valuable resource and a worthy read that falls somewhere between "popular science" and graduate-level text book.

It is easy to take Turing's work for granted as several decades of Computer Science education has solidified both our understanding and acceptance of this innovative work. At the time of publication, Turing's work was completely new, and this is somewhat reflected in the inconsistent use of notation in the paper. This unfortunately results in arguably the most important contribution to modern Computer Science being largely ignored by most undergraduate and graduate students. Most cull a basic understanding of the key results from standard "theory of computation" text books.

To a large extent, Petzold succeeds in bringing Turing's paper to life, and certainly provides a comprehensive running commentary that guides the reader through Turing's paper. It should be noted that while this is a well-written book that clearly achieves its goals in a clear and occasionally humorous fashion, it is in no sense meant for a general audience. Rather, the interested reader will most likely be the general Computer Science graduate student who is looking for an accessible, yet rigorous introduction to Turing's work. The book will also be a useful complement to a course in the theory of Computation, filling in many of the details that are often glossed over due to limits of time.

Review of[12]
Logicomix
Text by Apostolos Doxiadis and Christos Papadimitriou
Art by Alecos Papadatos and Annie di Donna
Published by Bloomsbury, 2009
314 pages, Softcover. Comic Book!

The Paradox of Price:
On Jan 5, 2010 Amazon listed it as $19.82 new, $67.81 used

Review by
William Gasarch `gasarch@cs.umd.edu`

Gasarch: Poincare, Hilbert, what did you think of the comic book *Logicomix*?

Poincare: I thought it was great!

Hilbert: I didn't like it!

Gasarch: Do you care to elaborate?

Poincare: It was intuitive! It told the story of Bertrand Russell's failed quest for the rigorous foundations of mathematics in a nice informal way! Framing the story as (a) Bertrand Russell giving a talk about *The Role of Logic in Human Affairs* where he actually talks about his own quest for complete rigor in mathematics, and (b) The creators of *Logicomix* discussing how to convey these ideas to the public, was brilliant! Track (a) reminds us that, in the past, there was a notion that perfect foundations for math and even non-mathematical things like politics, was possible! And Track (b) lets us know what the creators of this were thinking! Track (b) is also a natural example of self-reference!

Hilbert: But all works of mathematics need a firm foundation! This, this, ... comic book is too informal! It didn't define all of their terms! There were even some episodes in the book that didn't happen, like Russell meeting Cantor!

Poincare: How do you know that Russell never met Cantor?

Hilbert: The appendix of *Logicomix* said that the book was not meant to be history and pointed out some of the liberties they took!

Poincare: Then how can you complain? *Logicomix* is what it claims to be! As a logician you should appreciate that! Also, because they take liberties they are allowed to show the reader Cantor, Frege, Godel, and even us! And when I say *show* I mean *show* since this is a comic book! They are taking poetic— uh, ..., no, they are taking *comic license!*

Hilbert: AH- that is ambiguous! Do you mean *Comic* as in funny or *Comic* as in *Comic Book*!

Poincare: You know that I mean *Comic* as in *Comic Book*! It was clear from context!

Hilbert: And another thing, did they really need to tell us about Russell's personal life including his childhood and his many marriages!

Poincare: Even though it was not mathematics, it was interesting! It also reminds us that he was a real person with real problems!

Hilbert: I also didn't like the allusion that many people working in foundations are crazy! That charge does not really stand up to rigorous scrutiny! Cantor and Godel were not playing with a complete axiom set, but the rest were, at worst, eccentric!

Poincare: We can let the historians debate that one; however, it was worth bringing up!

Hilbert: I did not appreciate the pictures of us on page 145! Surely you didn't like being quoted as saying *Set Theory is a disease from which Mathematics must be cured!* Especially since you have been proven wrong!

Poincare: I found it delightful! And there is still time for Mathematics to recover! See Doren Zeilberger's Opinion 68! (http://math.rutgers.edu/ zeilberg/OPINIONS.html) And surely you did not like being quoted as saying, on page 152, *In Mathematics there is no Ignorabimus* (Ignorabimus means *we shall not know*)! You have been proven wrong by Godel and later Cohen! We shall never know if CH is true!

Hilbert: Alas you are right about that! But Set Theory is here to Stay!

Gasarch: Lets wrap this up! I want each of you two name one praise and one criticism of the book! And be brief!

Poincare: *Praise:* the fictional meetings between characters were imaginative and were, of course, new!
Criticism: Since I am a practical man I would have liked to see the uses that Logic were put to later! And more generally the birth of theoretical computer science! This may be quite a lot to do, so what I really want is a sequel! In the sequel perhaps Christos can be in both tracks!

Hilbert: *Praise:* The book took Russell's quest seriously!
Criticism: The book left out ZFC and the debate between Constructivists and the Formalists and the Platonist! This debate also drove some of the work on foundations!

Poincare: How about you Dr. Gasarch? Can you give us a praise and a criticism?

Gasarch: *Praise:* Seeing Christos Papadimitriou as a comic book figure! Seeing him drawn and talking captures him far better than a mere photo can! Possibly better than even a video!
Criticism: I have no criticism of the book, but I will give one of this review! Recall that *The Mathematical Coloring Book* was written in the form of a memoir (*When I first met Paul Erdos* ...)! Hence the review was in the form of a memoir (*When I first read this book* ...)! Since *Logicomix* is a comic book, I would have wanted the review to be a comic book! But alas, I have neither the time, money, nor expertise to pull that off! I hope that writing the review as a conversation where every sentence ends with an exclamation point or question mark will suffice!!

Hilbert: Your review is incomplete!

Gasarch: According to Godel that is inevitable (Poincare laughs, Hilbert does not)! Ahem, yes, I always end with an opinion as to who would benefit from the book under review! The reader does not need to know that much math! Knowing *about* math is more important then knowing any particular math! A bright high school student would benefit from it! However, even someone who has a PhD in logic would benefit from it! It is easy to forget our roots and this book takes us back to them! Back to a world when people thought certainty was possible!

Review of[13]
Proof and Other Dilemmas: Mathematics and Philosophy
Edited by Bonnie Gold & Roger A. Simons
Spectrum Series, MAA, 2008
346 pages, Hardcover, $46.00 new, $38.00 used, on Amazon
MAA prices: MAA members $43.50, for nonembers $55.95

Review by
Christopher Pincock (pincock@purdue.edu)
Department of Philosophy, Purdue University, West Lafayette, IN 47907

1 Introduction

This book is made up of 16 newly commissioned essays by prominent philosophers of mathematics along with some mathematicians and mathematics educators who have reflected on philosophical aspects of mathematics. As Bonnie Gold, one of the editors, explains in her introduction, the point of the volume is "to increase the level of interest among mathematicians in the philosophy of mathematics" (p. xiii). This goal seems to have led the editors to shy away from contributions that focus on the usual foundational debates such as the the debates between logicism, formalism and intuitionism from the first half of the twentieth-century. Also missing is much discussion of the ongoing work on logic and set theory that is often deployed in philosophical discussions of the nature of mathematics. Instead the editors focused on commissioning articles that would be accessible to mathematicians and advanced students of mathematics and that would also hopefully engage their interests. Most contributors responded by summarizing a central problem in the philosophy of mathematics, typically with an emphasis on their preferred take on the issue. The result is a very useful attempt to open more of a dialogue between philosophers and mathematicians.

2 Summary

The first part of the book is "Proof and How it is Changing". It contains articles by the philosopher Michael Detlefsen and the mathematicians Jonathan Borwein and Joseph Auslander. Auslander takes up the traditional position that deductive proof is a central feature of mathematics, but also admits that what counts as a proof may change over time as the mathematical community responds to mathematical and non-mathematical developments. He is also keen to emphasize that proofs have many functions beyond simply justifying a belief in a given theorem. These additional functions include providing an explanation of a theorem and helping to explore a new domain. Auslander goes on to criticize Zeilberger's call for a new "semi-rigorous mathematical culture" as "quite wrongheaded" (p. 71). This sets up an important contrast with Borwein's discussion which champions an "experimental mathematics" where computer-assisted exploration of mathematical results takes a central place. Borwein gives a list of eight roles for computers in mathematics which include not only "confirming analytically derived results", but also "exploring a possible result to see if it merits formal proof" and "discovering new facts, patterns and relationships" (pp. 44-45). These different roles are then illustrated with several intriguing examples.

Detlefsen takes up the place of proof in mathematics in his contribution, but in a way that reflects the relative priorities of philosophers as compared to mathematicians. This is reflected in Detlefsen's focus on clarifying the claim that computers are changing the nature of proof. He does this mainly by engaging with the arguments of other philosophers. He concludes that these arguments have not been successful and so it is premature to see some sort of revolutionary shift in our conception of mathematics. Similarly cautious results are obtained from Detlefsen's discussion of the role of diagrams in proof. While he concedes that some "insightful cases have been made for the significance of diagrammatic reasoning as justificative", it remains the case that "our understanding of possible limits on justificative uses of diagrammatic reasoning have been similarly advanced" (p. 27). The contrast here between the philosopher and the mathematicians runs throughout the volume. Most philosophers place a premium on clarifying a question and considering the pros and cons of the arguments that answer it. Mathematicians, at least as reflected in this volume, seem more eager to marshal examples that they think clearly favor their respective positions.

This dichotomy is especially prominent in the second part of the book focusing on social constructivist approaches to mathematics. The philosopher Julian Cole spends most of his article motivating a specific version of a social constructivist interpretation of mathematics according to which human actions bring abstract mathematical entities into existence. This position is hardly ever discussed in contemporary philosophy of mathematics despite its apparent popularity among some mathematicians. Cole complains, though, that Ernest's book defending social constructivism seems to have "no argument" and that the two arguments he can find in Hersh's discussion are not convincing (pp. 121-122). This leads Cole to offer an argument that his version of social constructivism is superior to its main competitor, namely platonism. Platonism posits abstract objects which exist independently of human actions, but Cole complains that "Platonistically construed mathematical domains are explanatorily and justificationally superfluous. Consequently, we should not accept their existence" (p. 125).

The mathematicians Philip Davis and Reuben Hersh provide articles that also support their respective versions of social constructivism. Davis's contribution considers the question "When Is a Problem Solved?" His answer is basically that a problem is never really solved: "meaning is dynamic and ongoing and there is no finality in the creation, formulation and solution of problems, despite our constant efforts to create order in the world" (p. 83). However, this somewhat pessimistic opening position is further refined as Davis considers several different kinds of mathematical problems and how a mathematician may decide that they have been more or less solved, at least for present purposes. Hersh's essay begins with the bold assertion that "Mathematical entities do exist, they are cultural items" (p. 96). This is followed by an investigation of how we might distinguish mathematics from other domains. Hersh concludes that the objects of mathematics are "all those abstractions that lend themselves to conclusive, irresistible reasoning" (p. 100). He then argues that the empirical study of humans using "history, sociology, anthropology, psychology, cognitive and neuroscience" (p. 101) is necessary to better appreciate what these abstractions are and how such conclusive reasoning is possible.

Philosophers dominate part three of the volume focused on the nature of mathematical objects. A review of the four essays by Charles Chihara, Stewart Shapiro, Mark Balaguer and Øystein Linnebo reveals not only substantial disagreement about the correct answer to this question, but also what the question is in the first place. For example, Chihara presents a nominalist interpretation of mathematics that accepts mathematical practice without positing the existence of any abstract objects. He offers a "structural account" of mathematics according to which mathematical axioms and theorems are not presented as true, but only as characterizing a model in which they are true. A benefit of this approach "is that we can avoid having to justify any analysis of what the assertions of the mathematical theories truly mean" (p. 144) and we can explain why "mathematical practice is simply not concerned with reference" (p. 154). By contrast, Balaguer insists that the main aim of a philosopher of mathematics is to construct a semantic theory which "is a presumably empirical theory about what certain expressions mean (or refer to) in ordinary discourse" (p. 181). This leads Balaguer to a fairly comprehensive survey of the different ways in which nominalists and platonists have tried to provide a semantic interpretation of mathematical language, although of course Chihara's non-semantic proposal is missing. In the end Balaguer argues for the somewhat counterintuitive conclusion that there is exactly one defensible version of nominalism and platonism, but that "there is actually no fact of matter" (p. 201) about which is correct.

In different ways, Shapiro and Linnebo develop non-standard interpretations of the nature of mathematical objects like the natural numbers 1, 2, 3, ... Shapiro, perhaps the most influential philosopher of mathematics working today, defends an "ante rem structuralist" interpretation of mathematics which posits the existence of abstract structures, like the natural number structure, that are metaphysically prior to the positions occurring in the structure, like the particular natural numbers (p. 173). A strong consideration in favor of this kind of structuralism, as against ordinary platonism, is that it can account for the way in which mathematicians seem to treat different set-theoretic structures as "the" natural numbers. Linnebo arrives at a very different understanding of the natural numbers by focusing on the family of systems of numerals which are adequate to represent the natural numbers. Linnebo argues first that we must permit significant differences in our understanding of reference to abstract objects and physical objects. For the natural numbers, this reference involves systems of numerals. The parallel between the numerals and the numbers is so tight that "whenever a natural number n possesses some arithmetical property, its doing so is inherited from the fact that the numerals that present n possess some related property" (p. 215). This leads Linnebo to conclude that even though there is still a sense in which numbers exist and numerals refer to numbers, the traditional problems for platonism can be overcome based on his understanding of the numeral-number link.

A highlight of the volume also comes in part three with the mathematician Barry Mazur's essay "When is One Thing Equal to Some Other Thing?". He offers one of the most accessible introductions that I am aware of to the basic idea that category theory can be used to characterize a mathematical domain independently of any particular choice for an underlying set theory. This sets the stage for an explanation of the tendency to treat isomorphic entities as equal. This point of view is illustrated using a characterization of the natural numbers as an initial object of what Mazur presents as the Peano category. This has advantages over a traditional presentation of the natural numbers as "it isolates, as Peano himself had done, the fundamental role of mere *succession* in the formulation of the natural numbers" (p. 232). This sort of approach to a mathematical domain is presented as a useful intermediate approach between an extreme "bureau-of-standards kind of definition" which arbitrarily selects one progression as the natural numbers and the alternative "Fregean universal quantification approach" (p. 232) which tries to define the natural numbers by quantifying over absolutely everything.

The final part of the volume has five essays which aim to either treat mathematics as a whole or else investigate the relationship between mathematics and its applications. The mathematician R. S. D. Thomas argues that mathematics is best understood "as sitting at the extreme of a spectrum of sciences" (p. 249) where the special focus of mathematics is the relations between things as opposed to their non-relational intrinsic nature. Thomas bases his proposal partly on a historical discussion of how mathematics has changed over time, and also on a consideration of the priorities of some current areas of mathematical research. Keith Devlin's aim in his essay is instead to investigate "What Will Count as Mathematics in 2100?". He argues that mathematics will be extended to incorporate patterns that are found outside the traditional domains of mathematics and its application. These include the notion of utility, Bayesian confirmation theory, theories of linguistic communication and financial economics applications such as the Black-Scholes model for option pricing. All such areas are partly mathematical now, but Devlin claims that further study will isolate patterns that are sufficient to expand mathematics to cover new territory.

The philosopher Alan Hájek considers one of these areas with his survey of probability theory and its philosophical interpretation. Hájek helpfully distinguishes debates about the different axioms of formal probability theory from the more philosophical issue of the viable interpretations of probability theory. For the axioms, Hájek reviews the problems concerning the popular approach that takes unconditional probability as basic and conditional probability as defined. Five different interpretations of probability are considered, including the logical interpretation developed by Carnap and the more subjectivist interpretation associated with De Finetti and Ramsey.

The more general problem of the applicability of mathematics is analyzed by the philosopher Mark Steiner. Steiner contrasts the logical problem of making sense of how mathematics contributes to scientific reasoning with the empirical problem of relating different mathematical concepts to different underlying physical regularities. Focusing mainly on the empirical problem, Steiner argues that some ways in which mathematics is currently used in science defy any straightforward explanation. In particular, he considers the way in which "isotopic spin" is represented using vectors in complex vector spaces and manipulated using the $SU(2)$ group. After explaining this treatment for some calculations involving particles and their isotopic spin, he suggests that they may involve the "Pythagorean Principle" that "At the deepest level of description, physical systems which are mathematically equivalent are physically equivalent – and thus one can be transformed into the other" (p. 320). Here, then, is a link from the application of mathematics to deeper philosophical mysteries about the nature of the physical world.

Finally, the mathematics education professor Guershon Harel explains how a broad conception of what mathematics amounts to can inform a program for improving mathematics education. Harel claims that we should not only focus on the products, or "ways of understanding", of mathematics, but also the process or "ways of thinking", that students and mathematicians employ to arrive at these products. He outlines an approach to mathematics education that emphasizes the intellectual needs of students and relates these needs to the sorts of assignments that students should be asked to complete.

3 Opinion

Hopefully these brief summaries suggest how the editors have sought to link philosophy of mathematics more closely with the interests of mathematicians. There is certainly a need for more engagement between mathematics and the philosophy of mathematics and I believe that this volume marks a productive first step in this direction. It is worth briefly asking, though, what barriers there are to philosophy-mathematics interaction and whether this volume will do much to overcome them. As I have already emphasized, philosophers and mathematicians tend to approach a philosophical topic with different priorities. The mathematicians in this volume often emphasize examples and exciting developments within mathematics, while the philosophers spend most of their energy clarifying concepts and criticizing the arguments of other philosophers. When taken to extremes either approach can frustrate the members of another discipline. Philosophers rightly ask mathematicians to clarify and argue for their positions, while a mathematician may become impatient with endless reflection and debate. A related barrier is the different backgrounds that most philosophers and mathematicians have. Philosophers are typically trained through the careful study of their predecessors and are taught to seek out objections and counterexamples. While most philosophers of mathematics have an excellent understanding of foundational areas of mathematics like logic and set theory, for obvious reasons few have reached a level of specialization in any other area of mathematics. By contrast, most mathematicians will not have much of a background in philosophy and will be tempted to appeal to the most interesting examples from their own mathematics even if they are not accessible to philosophers, let alone many other mathematicians. I am happy to report that most of the philosophical and mathematical discussion in this volume should be fairly accessible to everyone, but this probably happened only because the editors were looking out for complexities that might put off the average reader. Finally, it would be a bit naive to ignore the substantial professional barriers that stand in the way of any substantial philosophy-mathematics collaboration. To put it bluntly, nobody should try to get tenure by publishing for a community outside their home discipline. That said, it is encouraging to see philosophers and mathematicians at least trying to engage each other's interests and I hope these efforts will be continued and expanded in the coming years.

Review of[14]
Essays in Constructive Mathematics
by Harold M. Edwards
Published by Springer, 2005
211 pages, Hardcover, $79.95

Review by S. C. Coutinho collier@impa.br

1 Introduction

Like so many of us, I knew very early that I wanted to be a scientist. But, for a long time, that meant a chemist or a physicist. It was only when I began to learn euclidean geometry that I finally realised how interesting mathematics could be. Soon I was buying all the popular books on mathematics that I could find at the local bookshops (and that I could afford under my monthly allowance!). At that time, the mid 1970s, many of those books put a lot of emphasis on the marvels of infinite sets, which led them directly to the clash between Hilbert and Brouwer that had happened some fifty years before.

The time was the 1920s, and the key point of the controversy was a difference of opinion on what it means for a mathematical object to *exist*. Hilbert defended the following point of view: if you can write down a set of axioms for something, and these axioms uniquely define an object, and are not subject to internal contradictions, then the object may be said to exist. This thesis was contested by L. E. J. Brouwer; for him a mathematical object could only be said to exist if it could be constructed in a finite number of steps.

Actually, the philosophical conundrum at the heart of the controversy goes back at least to Aristotle's rejection of a completed infinity. Thus, to Aristotle, when we say that there are infinitely many natural numbers, we are only stating that we can keep counting for as long as necessary; we are *not* saying that there is a certain set with an infinite quantity of numbers. Cantor, of course, rejected this point of view and freely talked of infinite sets as objects complete in themselves. Since we cannot construct *infinitely many* numbers in a *finite* number of steps, constructivists reject Cantorian set theory. This, however, is a step that few mathematicians are prepared to take: in Hilbert's often quoted words "no one shall expel us from the Paradise that Cantor has created." Let us not forget that computer science itself has a foot in "Cantor's paradise"; just think of all the instances where a Cantorian diagonal argument is used.

Judging from those books I was reading some 30 years ago, axiomatics had come more or less victorious from this controvery; the "more or less" having to do with Gödel's theorems. This still may seem true nowadays for, if you say "constructivism" in a free association game with a practicing mathematician, the response will probably be "non-standard analysis", but will not go far beyond that. However, as Edwards points out in the preface of his book, constructivism is very much alive, more so now than it seemed to be 20 years ago. The reason for that may be summed up in one word: computer.

At first, the reason for this last comment may seem obvious: how could one deal with an infinite structure in a computer, anyway? However, there is more to it than meets the eye. To see that, let us consider a concrete example. Suppose we want to integrate a rational function f (a quotient of two polynomials) with integer coefficients. We learn to do it in calculus by computing the partial fraction decomposition of f and integrating term by term. The glitch, however, is that one must first factorize the denominator of f over the reals, because the numbers that come up in the partial fraction decomposition depend on the coefficients of the factors of the denominator of f. Unfortunately, real numbers cannot be exactly represented in a computer, so we have to accept an approximate answer. But, do we?

Indeed, we do not, for most general purpose computer algebra systems will now give an *exact* answer for the primitive of f. The way they get around the problem of representing real numbers is to do it in an implicit way. All these numbers satisfy some polynomial equation with integer coefficients (they are all algebraic numbers). So the system writes the solution in terms of the roots of a polynomial p of which all these numbers are zeroes, and outputs p together with the primitive of f. Since p has integer coefficients, the solution is exact.

This may seem cheating to you, if you have not seen it before; but it isn't. If you have p you can use constructive algebraic methods to recover the smallest subfield of $\mathbb{R}$ that contains all the numbers that appear as coefficients in the primitive of f. You can easily compute with them, if you need to, and the computations will be exact. The philosophy behind this may be summarised as follows. Instead of buying the whole of $\mathbb{R}$ in order to do the computation, you buy only those few real numbers you need as you go along. Since these numbers are algebraic, they admit a finite representation, which is exactly what you wanted.

2 Review of the book

The book under review presents several important topics in mathematics from a constructivist point of view. It is divided into five parts, the first four of which correspond roughly to the main topics covered in the book. The final, fifth part, is called "miscellany" and touches on several topics of a minor nature. Each one of the five parts is subdivided into sections or *essays*.

Part 1, called "a fundamental theorem" elaborates on the philosophy discussed at the end of the previous section, which reigns supreme throughout much of the book. Edwards's thesis is that the name "fundamental theorem of algebra" (a polynomial with complex coefficients always has a complex root) is a misnomer for, as he points out, it is not even a theorem in algebra. He suggests that a better candidate for that name would be the fact that every polynomial p with rational coefficients has a splitting field, which is the smallest field that contains $\mathbb{Q}$ and all the roots of p. Part 1 is dedicated to an algorithm that can be used to construct the splitting field of a given polynomial (with integer coefficients). I should point out, right away, that this book deals only with the thesis that *there are* constructive methods to do such and such; the algorithms are not intended for implementation and many of them would perform very poorly if one tried to run them in a computer.

The five sections of Part 2 deal mainly with Galois theory. Thus, essay 2.1 contains a proof of the fundamental theorem of Galois theory (as stated by Galois) and essay 2.4 is concerned with the computation of the splitting field of a generic polynomial of degree n, and a proof that its Galois group is the symmetric group in n symbols. The subject of Part 3 takes us back in time to the mathematics of Ancient Greece and of 7th century India; more exactly, the solution of diophantine equations of the form $Ax^2 + B = y^2$, which Edwards writes as $A\square + B = \square$. In order to solve these equations, he works with numbers of the form $e + g\sqrt{A}$ (which he calls *hypernumbers*) and their *modules*, by which he means a list of hypernumbers that are used to define a congruence relation. This part is inspired by Gauss's proof of quadratic reciprocity in Section 5 of *Disquisitiones*

Arithmeticae. A version of this proof, recast in terms of hypernumbers and their modules is given in essay 3.5.

Now we come to Part 4, which is my favourite by far. It concerns the computation of the genus of an algebraic plane curve defined by a polynomial with integer coefficients. This time the ideas stem from a memoir of Abel. After a general introduction (essay 4.1), followed by a discussion of the special case of cubics (essay 4.2), Edwards gives an algebraic definition of the genus (essay 4.3), and shows that it is amenable to direct calculation by an algorithm (essay 4.5) that uses Newton's polygon (essay 4.4). This part also contains a proof of the Riemann-Roch Theorem (essay 4.7) and a proof that the genus (as defined in the book) is a birational invariant.

The final part is deservedly called *Miscellany* and contains essays dealing with the (usually, so called) fundamental theorem of algebra, a discussion of proofs by contradiction in constructive mathematics (with a proof of the Sylow Theorems), and some topics of linear algebra. The very last essay is a review of the biography of L. Kronecker contained in chapter 25 of E. T. Bell's well-known book *Men of Mathematics*. Perhaps this essay would be better described as a defence of Kronecker, whose rôle as a critic of the views of Cantor and Weierstrass on the use of the infinite in mathematics is often stressed in detriment of his very important mathematical work. In fact, this criticism is sometimes said to have been so vicious and personal that it was partly responsible for driving Cantor to madness. This defence is indeed very proper, for Kronecker is a sort of "tutelary deity" of this book. Indeed, as Edwards explains at the end of the preface, he chose to call the sections of his book *essays* because "they essay to reopen the Kroneckerian road not taken". By that he means the kind of constructive mathematics, promoted by Kronecker, and more or less ignored by mathematicians for most of the 20th century.

3 Opinion

This book is a delight to read. To begin with it is written in clear English, and formulae are used only when truly necessary. Many arguments that in another book would require a handful of symbols are spelt out in clear prose, with no symbols in sight, and most readers will thank the author for that, as it definitely furthers understanding. Moreover, the required background is kept to a minimum, so the book can be read by anyone with a good understaning of basic algebra. The many explicit examples calculated throughout the book will definitely help in that respect.

The style of the book itself is sufficient reason for reading it. Many mathematics' books nowadays give the impression that their authors are like oracles, merely putting on paper the mathematics that is written in "the book"–to borrow Erdös expression. Such books hide the fact that mathematics is a human activity and make it look like it were written on stone. No such nonsense will be found in this book. As the author points out, the word *essay* is closely tied with the idea that what is expressed therein is "the personal views of the author". And it couldn't be otherwise, for the whole book is a defence of the author's point of view–not shared by many mathematicians today–that constructivism is very much worth pursuing. Moreover, this defence of constructivism is not achieved by a philosophical discussion, but by actually doing mathematics in a constructive way. Of course this means that one is bound to disagree with the author every so often, but that only makes reading the book even more worthwhile.

I cannot finish without calling attention to two very nice features of the book. First, since he is most often talking about algorithms, Edwards revives the old Euclidean usage of stating a theorem in the form "construct such and such", and giving the description of the construction as its proof.

In this respect (as in many others) his example deserves to be followed by other authors, especially those of books that are mostly concerned with algorithms. The second feature is the way that the history of mathematics is used in the book; although it might be better to talk of *the great classic works of the past*, rather than of history of mathematics. For Edwards uses these classics as a source of inspiration for doing mathematics and not merely for the sake of a knowledge of the history of his subject. One would wish that more mathematicians read the masters in search of inspiration, instead of merely following the most recent fads.

Review of[15]
Is Mathematics Inevitable? A Miscellany
by Underwood Dudley (editor)
The Mathematical Association of America, 2008
324 pages, Hardcover
MAA prices: For members \$45.50, for nonmembers \$56.95
Amazon prices: New \$51.53, Used \$41.49

Review by
José de Oliveira Guimarães, josedeoliveiraguimaraes@gmail.com

1 Introduction

The book is, as the title says, a miscellany of articles on Mathematics chosen by the editor. It seems a small version of the four book set "The World of Mathematics" by James R. Newman. It contains twenty six articles treating subjects like the history of math problems, biographies of mathematicians, probabilities, and a teaching method. Each article is preceded by a commentary from the editor ranging from fifteen lines to two pages. This explanation puts the article that follows in context and gives the necessary historical background to understand it. The articles themselves range from one and a half to fifteen pages.

2 Summary

The book articles can be divided roughly in six categories: 1) comments on Mathematics and mathematicians (four) 2) Mathematics (six) 3) history (ten) 4) teaching (one) 5) humor (one) and simply 6) miscellany (four). In the first category we find an article by Dieudonné on Mathematics and mathematicians explaining his view on the subject, mathematical research, history, and so on. Another article of this category, by Morris Kline, explains the nature of Mathematics. In category 2, on Mathematics, we found a very interesting content. There are two articles on statistics and an article on patterns of sequences of numbers (mainly) in which the reader is invited to answer if each pattern is correct or not (35 patterns are presented, with answers). The text "A Parallel Postulate" presents a historical perspective of the parallel postulate of Euclid including dozens of axioms logically equivalent to it.

There are ten articles in category 3, history, including one on the origins of some popular math problems and one on calculus that appeared in the first editions of the Encyclopedia Britannica.

Both are very interesting. However, some of the texts are less charming, such as the one on the life of the mathematician John Pell. It was written in the end of the seventeen century by John Aubry in an old English style (although the spelling was modernized). Aubry describes many details of Pell's life but few related to mathematics. There is also the autobiography of Cardano that is historically fascinating. Category 4, teaching, has one article on the Moore method by Paul Halmos which is a teaching method in which the students must learn by themselves (the teacher gives some definitions and few explanations and the students must prove the theorems by themselves). It is a hard version of PBL (problem based learning). There is one article on humor which proves that the number of legs of a horse is infinite.

Finally there is a miscellany category with four articles which probably will not be among the favorite ones for most people. Among these there is an article on calculating prodigies, people that can do astounding mental calculations. Although this is an interesting subject, the article describes details of the lives of two prodigies that are unrelated to calculation and mathematics . It is also included in this category a speech made by a member of the United Kingdom Parliament in defense of the teaching of quadratic equations. There is also an article by a crank called James Smith which claimed to have squared the circle. This may be historically interesting.

3 Opinion

This book should please anyone interested in mathematics and science. Most of it only demands knowledge of the Mathematics taught in high school. The book shows several faces of Mathematics — the articles encompass history, teaching, humor, and technical material. Although there are a few articles with too many details on non-attractive subjects, I guess most people will enjoy reading most of them.

Review Of[16]

A Concise Introduction To Languages and Machines

Author of book: Alan P. Parkes

Publisher: Springer 2008, $29.95

Reviewer: Mike Williams `mwilliams@google.net`

1 Overview

This book is part of the "Undergraduate Topics in Computer Science Series". It covers material suitable for an undergraduate course in formal languages, and doesn't really assume much in the way of prior knowledge.

2 Summary of Contents

2.1 Part 1: Languages and Machines

Part 1 of the book focuses on defining formal languages, and abstract machines that can be used to recognize them.

Chapter 1 “Introduction”, is a short chapter that describes the format of the book, and sets the tone for the rest of it. The author relates his own struggles to understand the material when he was a student.

Chapter 2 “Elements of Formal Languages” defines many elements of formal languages, such as alphabets and strings, and introduces some standard notation that is used later in the book. It also defines recognizers and generators for languages, and the Chomsky hierarchy. There is a simple examples of each type of language, as well as illustrations showing how sentences of a formal language can be parsed.

Chapter 3 “Syntax, Semantics and Ambiguity”, introduces derivation trees, and shows some examples of how they can be constructed. Ambiguity is defined, and illustrated nicely with some clear examples.

Chapter 4 “Regular Languages and Finite State Recognizers”, focuses on regular languages, both as grammars and finite state automata/recognizers . The equivalence of both is demonstrated. There is a good discussion of non-determinism, as well as examples of converting non-deterministic finite state recognizers into deterministic ones. The author presents an informal argument to show that any non-deterministic recognizer can be converted into an equivalent deterministic recognizer. There is also a section describing minimal regular languages, and procedures for calculating minimal languages.

Chapter 5 “Context Free Languages and Pushdown Recognizers” introduces context free languages, and has an interesting section describing how grammars can be transformed without changing the language they generate. Push-down recognizers are introduced, along with some simple and well-illustrated examples of how they work. Non-deterministic and deterministic languages are compared, and they are shown to not be equivalent.

Chapter 6 “Important Features of Regular and Context Free Languages” is one of the more mathematically interesting chapters; it discusses some of the closure properties of formal languages, and it also formally proves the hierarchical relationship between different classes of languages. The Pumping Lemma (although the author doesn't call it that) is used to show that some languages can be shown to be not regular (or context free, depending on the flavor of the Lemma). The explanations are clear and there are some helpful illustrations.

Chapter 7 “Phrase Structure Languages and Turing Machines” introduces Turing machines and uses some arguments to show that they are at least as powerful as the other languages introduced so far by showing that they can simulate finite state recognizers and pushdown recognizers. There are also some complete descriptions of Turing machines for some of the simpler languages to show both how they work and also to convince the reader that they are capable of recognizing more complex languages, such as ones that cannot be recognized by the earlier machines. Decidability and acceptability of languages are also introduced, but not in much detail.

2.2 Part 2: Machines and Computation

Part 2 of the book focuses more on how the abstract machines introduced in part 1 can compute interesting results.

Chapter 8 “Finite State Transducers” returns to regular languages, and it shows how machines can be used to produce output instead of just accepting or rejecting strings. There are several example transducers presented that do simple things like add 2 numbers or multiply a number by a constant. The exact details of the machines are presented, which is particularly interesting since it gives not only a good example for how to construct other transducers, but it also convinces the reader that even seemingly-complicated problems can be solved by simple machines (simple in terms of the their operations, even if it takes many of them to get the job done). Some of the limitations of regular languages and finite state transducers are introduced.

Chapter 9 “Turing Machines as Computers” is one of the most most interesting chapters in the book. It discusses how Turing machines can be used to perform simple arithmetic and logical operations, and it presents very detailed descriptions (down to the level of tape moves and read/write operations) for addition, subtraction, and multiplication operations; division is sketched out. Simple logical operations are also presented. Much like the previous chapter, the details do a good job of convincing the reader that any algorithmic operation can be performed by a Turing machine, even if the exact details of the operations are tedious.

Chapter 10 “Turing's Thesis and the Universality of the Turing Machine” introduces the concept of universal Turing machines. Encoding of various inputs and machine representations are described, and Turing's thesis about the power (and limitations) of the machines is presented. Extensions to the standard Turing machine in the form of multiple tapes, and non-determinism, are described, and there are procedures for converting the extended machines into standard form.

Chapter 11 “Computability, Solvability and the Halting Problem” returns to decidability, and presents the Halting Problem. There is a good illustration of the Halting Problem and the implications of its undecidability. Computable languages are defined, and examples of a non-computable and non-decidable languages are shown.

Chapter 12 “Dimensions of Computation” briefly introduces complexity theory. Non-determinism is presented as a form of parallel computation and the implications on running time are discussed. Big “O” notation is presented, and there are also examples of algorithms in various complexity classes. The book finishes with the open “P=?NP” question. This chapter seems a bit out of place with the rest of the book.

3 Style

The book is well-written, with very few typos. The author interjects an occasional aside or a bit of encouragement, which can be helpful in easing students into what can normally be fairly dry material. There is some formalism in the presentation, but little in the way of serious proofs. It would be a challenging but approachable book for a non-computer scientist, but students with formal mathematical backgrounds might be a bit disappointed by the lack of rigor.

The examples presented in the text are clear and they support the main text well. There are line-diagrams illustrating some of the machines presented in the book; for the most part, they are well-designed, clear and helpful. The author uses the Pascal language in several examples; while it isn't necessary to know the language to understand the examples, the book might be a bit more approachable if it used a more mainstream language.

There are a few problems at the end of each chapter; most of them are thought problems and they aren't particularly difficult, although there are a few that could make interesting programming assignments. Some of the problems have solutions at the back of the book, and they are fairly thorough and helpful. The "Related Reading" section has some interesting titles, some of which offer more formal mathematical treatments of the subject matter (which could be very helpful given the informal treatment in this book).

The index and table of contents are fairly extensive and well ordered.

4 Opinion

"A Concise Introduction To Languages And Machines" would be a good introductory book for an undergraduate course in computation, or as a reference book in a course on compiler design or language processing. The lack of rigor in some of the chapters would prevent it from being very useful for more advanced or graduate courses. It would be suitable for motivated self-learners who are interested in the subject matter, or as a background book for a student specializing in other areas. The price is very reasonable, but its also available free on-line.

Review of
A Second Course in Formal Languages and Automata Theory [17]
Author: Jeffrey Shallit
Publisher: Cambridge University Press, 2008
ISBN 978-0-521-86572-2, $48.00

Reviewer: Kevin A. Wilson (kwilson@rti.org)

1 Overview

The inclusion of a course in automata, computability, and complexity theory is standard in most Computer Science curricula. Such a course typically introduces fundamental theoretical computer science concepts including several formal models of computation and their associated languages. Most often included are finite state automata and regular languages, push-down automata and context-free grammars, and turing machines. Along the way, concepts such as various closure properties of languages, what it means for a problem to be "decidable", and proof that there exists a countably infinite number of problems that can be solved on a computer, are presented. Most courses also include a section on complexity theory, which usually focuses on determining whether a given language is part of the NP-Complete sub-class of recursive languages. Unfortunately, it is often the case that such a course forms the only theoretical computer science education a student receives, either at the undergraduate or graduate level.

Shallit's book is an effort to fill this void by providing a *second course* that explores some of the more advanced topics of automata theory such as combinatorics, generalizations of finite-state automata (finite-state transducers, two-way FSA's, transformation automata), minimization and state complexity. Advanced closure properties are discussed for both regular languages and context-free languages, and additional language classes (context-sensitive) and the Chomsky hierarchy are also presented.

2 Review

The book opens with a brief recapitulation of topics typically covered in a theory of computing class, primarily for the purpose of establishing consistent notation and a common background for the subsequent chapters. Topics covered in Chapter 1 include a review of basic set theory and associated properties (union, intersection, complement, cardinality and the empty set). Basic definitions of words, strings, and alphabets are reviewed prior to discussion of finite state automata and regular languages. The pumping lemma for regular languages is briefly presented followed by context-free grammars and pushdown automata. There is a review of closure properties for the various classes of languages, turing machines, and a discussion of the universal turing machine. The chapter concludes with a review of computability theory (decidable and undecidable languages), and a brief introduction to NP-Completeness via proof of 3-Sat, and Savitch's theorem. As might be expected, presentation of this material is quite terse as the book is clearly aimed at those who have already completed a *first course* in formal languages. Nevertheless, the presentation is clear and serves its purpose well.

Chapter 2 is a continuation of the preparatory work and presents various topics related to combinatorics on words. Topics such as infinite strings from a finite alphabet, concatenation of finite and infinite strings, and periodic strings are introduced along with a detailed exposition of string morphisms (i.e. a map from Σ^* to Δ^* the obeys that identity $h(xy) = h(x)h(y)$.) For example, for $\Sigma = \{m, o, s, e\}$ and $\Delta = \{\epsilon, a, n, t, l, e, r, s\}$, the following defines a morphism:

$$\begin{gathered} \text{h(m)} = \text{ant} \\ \text{h(o)} = \epsilon \\ \text{h(s)} = \text{ler} \\ \text{h(e)} = \text{s} \end{gathered}$$

We can see that the above defines a morphism because $h(moose) = antlers$. This leads nicely into Levi's lemma and the Lyndon-Schűtzenberger theorem which describes strings that have an overlapping sequence that could be partitioned with one side or other of the string. The example provided is "alfalfa" - the middle "a" can be attached to the first half of the string, or to the second, forming the word "alfa" in both cases. Definitions of word conjugates (e.g. $u = en, v = list, uv =$

listen, $vu = enlist$) and repetitions in strings are also provided in preparation for the concepts discussed in the remainder of the book.

Chapter 3 is, in my opinion, the pivotal chapter in the book, building on the first two chapters and covering a lot of ground. Topics in this chapter all form logical extensions of work covered over the course of a typical theory class. The primary focus is on various generalizations of finite-state automata, beginning with a detailed discussion of Mealy and Moore machines – two variations of DFA's that return strings (from states in Moore machines, and from transitions in Mealy machines). Equivalence of these machines is proven, which themselves form precursor to transducers - non-deterministic Mealy machines with finite states and transitions labeled within input and output strings. Additional closure properties and operations on regular languages are presented and proven, along with the definition of two-way finite automata, another generalization of FSA's that do not add more power. The chapter concludes with transformation automata, proof of the Myhill-Nerode theorem - an advanced method used for proving a language is non-regular, and state complexity.

Chapters 4 and 5 cover advanced properties and parsing of context-free grammars. Building on chapter 3, advanced closure properties (context-free languages are closed under substitution by context-free languages, morphism, and inverse morphism) are discussed along with unary context-free languages, and the use of Ogden's lemma to prove that a language is inherently ambiguous. The "Interchange-lemma" is presented as an advanced method of proving that a language is not context free. Parikh's theorem is presented as a generalization of an earlier result that a unary language is context free if, and only if, it is regular. Chapter 4 concludes with deterministic context-free languages and linear languages. Chapter 5 presents various standard parsing algorithms including CYK and Earley's algorithms (both of which are polynomial time, $O(n^3)$, parsing algorithms for grammars in Chomsky Normal Form), and two more generalized parsing algorithms: top-down, LL(k), and bottom-up, LR(k). The chapter concludes with various properties of these algorithms.

Chapter 6 follows logically from chapter 5 and discusses unrestricted grammars, i.e. those where both the left and right-hand side can contain a combination of variables and terminals. Formal properties of these languages are discussed, such as the fact that a language defined by an unrestricted grammar is recursively enumerable, and there exists an unrestricted grammar for every recursively enumerable language. Some more advanced topics relating to the compressibility of strings are also included (Kolmogorov complexity, and the Incompressibility method) and the chapter finishes with extensive coverage of the "busy beaver problem", Post's correspondence problem, undecidability and complexity of regular and context-free languages. In particular the language L_{regex} is shown to be *PSPACE-Complete*.

The book concludes with discussion of several other classes of languages including those defined by a linear-bounded automaton (context-sensitive languages), the Chomsky hierachy, two-way deterministic PDA's and Cooks's theorem.

3 Opinion

This is a graduate level text book that understandably assumes a significant amount of prior knowledge, and indeed interest in the subject. It is aimed at those students who either need or want a more in-depth knowledge of formal computer science, and is largely successful in meeting its goals. While the review of topics in chapter 1 is very brief, and the reason for the deviation through combinatorics is not immediately obvious, as one progresses through the book, the logical order of the topics becomes clear.

The topics presented form the logical “next-steps” in each area. For example, finite-state transducers (FST’s) are covered in depth as a generalization/extension of finite-state automata. Students are often required to learn such topics when applying the basic principles learned in an introductory class. For instance, FST’s are a key theoretical mechanism used extensively in many applications, such as natural language processing or bioinformatics. Similarly, an introductory course rarely covers the parsing of context-free grammars, a technique required to apply the theory to real-life problems. Shallit’s book succeeds in presenting and explaining these important additional topics accurately and concisely, backed by rigorous proofs.

The book is well organized and the topics are well chosen, with many example problems for readers to attempt. In addition, at the end of each chapter a series of open research problems, suggested student projects, and general notes are provided. Unfortunately, no solutions are provided, and given that this is a book that may most often be used for self-study, some solutions would be a welcome addition. The information is presented clearly and concisely, and will be of interest to those looking for a more in-depth coverage of theoretical computer science and to those involved in the practical application of those principles in many areas.

Review of
Automata Theory with Modern Applications
Author of book: by James A. Anderson
Cambridge University Press, 2006, viii+256 pages
ISBN 0-521-84887-3 (Hardback), 0-521-61324-8 (Paperback)

Review by Kyriakos N. Sgarbas (sgarbas@upatras.gr)
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1 Overview

This is a book on the theory of automata and formal languages. Although according to the preface one of the purposes of the book is to be used as a textbook, it is rather concise (256 pages) and advances to some issues not normally mentioned in ordinary automata theory courses. It consists of seven chapters divided into three to ten sections each. Every section has its own set of exercises, although without solutions. The author mentions however that solutions to all exercises are available to the instructors upon request. The first five chapters contain nearly everything one could expect to find in a typical book on automata theory, from sets and functions to Turing machines and formal grammars. The last two chapters discuss more specialized issues, namely the visualization of languages in a two-dimensional grid and a special class of regular languages called splicing languages inspired by the splicing and recombination process of DNA strands in molecular biology. The style of the book is theoretical / mathematical and it would certainly be helpful for the reader if he has some background on discrete mathematics.

2 Summary of Contents

I suppose that the contents of the first five chapters of the book are more or less common knowledge to anyone reading this column, so I do not elaborate much on them. Some more details are given concerning chapters 6 and 7, where I try to explain (less formally and more practically) the basic new notions.

2.1 Chapter 1:

Introduction (22 pages) summarises the basic notions and properties of sets, relations, functions and semigroups. The definitions on this chapter include partitions, equivalence classes, chains, lattices, semilattices, monoids, submonoids, isomorphisms and homomorphisms.

2.2 Chapter 2:

Languages and codes (14 pages), despite its generic title this chapter mentions only regular languages, very briefly (in just 4 pages) and then extends to *codes* , *retracts*, and *semiretracts*. A *code* is defined as a set of sub-strings that combined and concatenated can produce every string of a language. A *retract* is the image of a function f such that $f(f(x)) = f(x)$, for all x in that image. An intersection of a finite number of retracts is called *semiretract*. The sections on retracts and semiretracts are marked "optional" in the book, since they are not required for comprehending the next chapters.

2.3 Chapter 3

Automata (77 pages) is the longest chapter of the book and includes nearly everything one could expect regarding automata: deterministic and non-deterministic automata, Kleene's Theorem, minimal deterministic automata, syntactic monoids, the Pumping Lemma for regular languages, decidability, pushdown automata, Mealy and Moore machines. However, I would expect at least some reference to transducers, especially since the term is later mentioned in Chapter 7.

2.4 Chapter 4

Grammars (55 pages) consists of four sections on formal grammars, Chomsky and Greibach normal forms for context free grammars, pushdown automata and their relation to context free languages, the Pumping Lemma and decidability.

2.5 Chapter 5

Turing machines(41 pages) discusses deterministic and non-deterministic Turing machines, their acceptance of context-free languages, the halting problem for Turing machines, and concludes with a section on undecidability problems for context-free languages.

2.6 Chapter 6

A visual approach to formal languages (21 pages) starts with word combinatorics, first introducing *powers* and *primitive* words. A word is called a *power* if it can be produced by the concatenation of the same sub-word, two or more times. A word is *primitive* if it cannot be expressed as a power of another word. The *spectrum* of a word is defined as a set of integers denoting which powers of the word belong to the language. Words with identical spectra form *spectral partitions*. The previous definitions are then used to provide a way to “visualize” or “sketch” a language on a two dimensional semi-plane, by ordering the words over the x-axis and drawing each word’s spectrum as a series of tiles over the y-semiaxis. The function determining this visualization is called the *sketch* function. Some more elaborate definitions follow on *sketch parameters* and *flag languages* and the chapter concludes with a discussion on sketching regular languages. Strangely enough there is not a single picture in this chapter. Even the visual representations of the examples are described verbally.

2.7 Chapter 7

From biopolymers to formal language theory (14 pages) starts with a very brief description of DNA and RNA sequences, the conventions used to model them as words over finite alphabets and the process of splicing DNA strands by proper enzymes to produce recombinant strands. Then a formal model for the splicing process is defined, introducing the notions of the *splicing rule* (i.e. a symbol pattern that triggers a splicing and determines the recombinant string) and the *splicing language* (i.e. a language containing all recombinant strings produced by successive splicing operations of an initial language, according to a set of splicing rules). It is explained that splicing languages are a subclass of regular languages. Then the special class of *reflective splicing languages* is defined (i.e. languages which their splicing rules allow the recombinant strings to be spliced again and recombined to their original form) and finally at the last section of the chapter it is proved that “it is algorithmically decidable whether a given regular language is a reflexive splicing.”

2.8 Appendices

There are also two appendices, Appendix A on Cardinality and Appendix B on the Co-compactness Lemma. Since the appendices are very short (less than 2 pages each) I suppose their contents could have been merged into some of the first chapters instead.

The book concludes with a short list of 37 references +7 more under the title *Further reading*, and an index of terms. Surprisingly, there is no mention to Aho, Ullman, Hopcroft or Chomsky in the references.

3 Opinion

First of all I have to admit that I found the title of the book somewhat misleading: *Automata Theory with Modern Applications*. In my mind *application* means the use of theory to solve a real-world problem. (I may be wrong, though.) Anyway, I was already aware of some common applications of automata, e.g. in compilers, in speech and natural language processing etc, and I was expecting to read about some new/modern ones. So I was rather disappointed after completing this book, because even for the new notions introduced at the last two chapters there was not any note concerning their application to real-world problems.

In fact, Chapter 6 was very interesting and enjoying to read, but there was no mention to where this visualization concept could be applied, how to be applied and what the expected benefit would be from its application. Only the last exercise of the chapter mentions some relation to Art (painting).

Chapter 7 although it starts very promisingly, explaining the structure of proteins, the DNA strands and their splicing process, it then shifts to theory, creates a mathematical model inspired by the physical process, builds on it heavily producing some quite interesting results and stops there. It neglects to shift back and apply (or at least compare) these theoretical results to the real world. For example, the fact that all splicing languages are regular is quite astonishing, but what is the impact of this observation on molecular biology? The author gives some references that evidently elaborate on these issues, but still a feeling of incompleteness remains after reading the chapter.

Other than that, the book is fine from a mathematical point of view and its first five chapters contain more or less everything that a standard course on automata theory and formal languages should cover. However, as a textbook, I would not be very eager to recommend it instead of a classic choice like Aho & Ullman's book [1], or some more extensive newer texts [2], [3], due to its compact writing and the limited number of examples, especially in the first two chapters.

But readers who are already familiar with the basic automata theory will enjoy the last two chapters that extend the basic theory on language sketches and splicing languages and researchers could be tempted/inspired to further explore these new attractive frontiers.

4 References

[1] J.E.Hopcroft, J.D.Ullman, "Introduction to Automata Theory, Languages, and Computation", Addison-Wesley Publishing Co., 1979.

[2] J.E.Hopcroft, R.Motwani, J.D.Ullman, "Introduction to Automata Theory, Languages, and Computation", 2nd ed., Addison-Wesley Publishing Co., 2001.

[3] A. Meduna, "Automata and Languages. Theory and Applications", Springer-Verlag, London, 2000.

[4] P.Clote, R.Backofen, "Computational Molecular Biology – An Introduction", Wiley, 2000.

Review of[18]
Change is Possible: Stories of Women and Minorities in Mathematics
by Patricia Clark Kenschaft
Published by AMS, 2005
212 pages, Softcover
Amazon: $25.00 new, $23.00 used
AMS price Member: $24.00, Nonmember: $30.00

1 Introduction

In "Change is Possible: Stories of Women and Minorities in Mathematics," Patricia Clark Kenschaft presents an extensive history of a selection of women and minority mathematicians. As evidenced by the title, Kenschaft takes a generally positive and forward-looking approach to the issue. The book includes mathematicians from the nineteenth century to the present and describes their experiences (within both their academic and non-professional lives) by way of facts, anecdotes, and the author's broader opinion of the implications of these experiences. The book is extensively researched from personal interviews with the mathematicians themselves or their relatives, colleagues, etc. One of the first such detailed chronicles, it does not attempt to make generalizations by picking representative examples, but instead provides full stories of the women and minorities profiled.

2 Summary

The book begins on a positive note, acknowledging the many mathematicians who are "good white men"[19] and have helped women and minorities to obtain education and jobs. In some cases, mathematics was ahead of other fields when it came to diversity in the nineteenth and early twentieth centuries. The author and some of the subjects of her book theorize that perhaps in some ways it was easier for women and minorities to break into mathematics than other fields since evaluation of their work was more objective. In fact, the first Ph.D. awarded to a woman in the nineteenth century was awarded in mathematics to Sofia Kovalevskaia in 1872.[20] But in some ways women's presence in mathematics decreased in the twentieth century. A higher percentage of women delivered papers at the American Mathematical Society meeting in 1905 than in 1971.[21] (Kenschaft does not hypothesize a specific reason for this, other than a general changing of the times and the presence in the early 1900s of a few prominent women mathematicians who served as mentors for many others.)

One of the restrictions that women mathematicians faced frequently in the twentieth century was due to nepotism laws in place at many universities.[22] A large number of highly qualified female mathematicians were ineligible for academic jobs because their husband worked in or joined that department. Josephine Mitchell, for example, was already a member of the faculty at the University

[19]Page 5.
[20]Page 6.
[21]Page 45.
[22]These laws were often applied only to the female member of the couple, and as such evidenced sexism and not just a desire to decrease nepotism.

of Illinois when her future husband was hired. Though they were hired independently and she had been there longer, when they got married the university did not renew her contract.[23]

While most of these official restrictions have been lifted and many more women have entered mathematics, barriers still remain for women who want to become academic mathematicians. For example, as of this book's writing in 2005, Harvard, the oldest institute for higher learning in this country, had never conferred tenure upon a female member of the mathematics faculty. This dubious distinction is no longer true. The past year saw Sophie Morel become the first tenured woman ever in the Harvard mathematics department, more than 370 years after the school's founding.[24]

Until the mid nineteenth century, access to education was minimal for African Americans (even for free black men) due to slavery. Despite this, some black mathematicians were recognized for their mathematical achievements even before the end of slavery. Benjamin Banneker published an almanac in the 1790s containing astronomical calculations marketed as "an extraordinary effort of genius."[25] After the abolition of slavery, discriminatory policies and economic barriers continued to limit access to higher education. The first African American to pursue doctoral studies in mathematics was Kelly Miller. At Johns Hopkins University he experienced intellectual isolation from his classmates due to his race. But it was a raise in tuition from $100 to $125 in 1888 that meant he was never able to graduate. The first black woman to earn a doctorate in mathematics, Euphemia Lofton Haynes, graduated from the Catholic University of America in 1943, 26 years after earning her bachelors from Smith College. She, like many women and minorities in mathematics, found a satisfying career within and outside of academia, teaching high school, college, and eventually serving as the Washington, DC Board of Education president.

Latinos, defined by Kenschaft in the spirit of underrepresentation as people of Latino ancestry born in the United States, have been prominent mathematicians since the mid twentieth century. In addition to racism and a lack of monetary resources contributing to the presence of proportionally fewer Latinos in mathematics higher education and academia, hiring policies at governmental organizations had a discriminatory effect on Latinos due to family ties abroad or settlement near an international border. William Vélez, now a Professor of Mathematics at the University of Arizona, recalls this story about applying for a job at the National Security Agency:

> At the exit interview, they told me that if I worked for them I couldn't have contact with foreign nationals. I told them, "I live on the border. You can't be serious about this." "No," they said. "Here's the rule. Are you willing to comply?"[26]

Many women, African Americans, and Latinos, having faced discrimination in their own schooling, serve as mentors and provide encouragement for students in their communities. One example that Kenschaft profiles, is that of a program for middle and high school students started in Texas by Manuel Berriozábal, a professor at the University of Texas, San Antonio. The Texas Prefreshman Engineering Program (`http://www.prep-usa.org`), founded in 1979, runs a summer math and

[23] Page 74.

[24] Harvard has had women fill a specially created position titled "Professor of the Practice." The tenured faculty with this designation tend to teach only topics up to and including Calculus. For more information about the current situation for women at Harvard, see http://www.nytimes.com/2010/03/06/education/06iht-ffharvard.html. Other elite U.S. institutions have had female full professors in mathematics departments since the nineteenth century, for example Susan Cunningham who was the head of the Swarthmore College mathematics department from 1869 to 1906.

[25] Page 82.

[26] Page 114.

science program with 80% minority students and a slight majority of female students. Longevity studies have shown that participants in the program are more likely to graduate college and to graduate with a major in the sciences.[27] Over 28,000 students have participated in this program.[28]

Kenschaft concludes with a discussion of the state of women and minorities in mathematics today. While more women and minority mathematicians succeed in academia now, and many groups have organized around their minority status to combat discrimination and advocate for themselves, there is reason for both hope and worry. The main problem that Kenschaft identifies for the future of minority mathematicians is that of the failing United States public school system at all levels. Increasing the number of women in mathematics is discussed in the context of broader reforms to education and academia including double-blind reviewing of conference and journal articles, greater access to affordable child care, and increased awareness of subtle discriminatory practices within classroom and departments.

3 Opinion

"Change is Possible: Stories of Women and Minorities in Mathematics" gives a detailed accounting of the lives of the mathematicians chronicled. Perhaps due to a desire to record as many of the facts as possible, the recounting is occasionally dry, in the style of a litany of generations. These passages are broken up with poignant, sometimes amusing, illustrative stories from the lives of these mathematicians. It is these vignettes that make up the heart of the book. While the author's own, generally hopeful, analysis of these anecdotes and trends is often insightful, the prose switch from factual recitation to story telling to analysis is not always smooth. Despite this, Kenschaft successfully conveys a sense of the issues and history of women and minorities in mathematics.

The main focus of "Change is Possible" is on mathematicians and not computer scientists. While, of course, computer science finds many of its roots in mathematics, it does have some significant differences when it comes to the state of women and minorities. As one example (not from the book), consider that women made up approximately 45% of undergraduate mathematics degrees awarded in 2006, while only approximately 20% of computer science degrees, and that while the percentage of women in mathematics has been increasing, the percentage of women in computer science has been decreasing since its high of approximately 35% in the 1980s.[29] For those wanting to understand the state of women and minorities in computer science today, Kenschaft helps to provide insight through historical context. As evidenced by Harvard's example, it could be instructive for each of us to examine the situation today at our own institution.[30] The percentage of women and minorities in the department is one important indicator, but consideration of issues

[27] Pages 128 – 129.

[28] http://www.prep-usa.org/portal/texprep/generaldetail.asp?ID=107

[29] The Association for Women in Science maintains statistics of this sort (http://www.awis.org/). For more data, including information on race, see the Taulbee survey for computer science (http://www.cra.org/resources/taulbee/) and the American Mathematical Society survey for mathematics (http://www.ams.org/employment/surveyreports.html).

[30] Unfortunately, the book does not contain an index, so finding the history of your institution requires a more detailed read or another source. For more information on African American mathematicians, consult information about Mathematicians of the African Diaspora at http://www.math.buffalo.edu/mad/. Other sources include the professional organizations the Association for Women in Mathematics (http://www.awm-math.org/), the National Association of Mathematicians (http://www.nam-math.org/), the Council for African-American Researchers in the Mathematical Sciences, and the SIAM Diversity Advisory Committee (http://www.siam.org/about/com_div.php).

such as maternity leave, department environment, and clear communication of policies are also important and benefit all members of the community. The interested computer scientist or one looking to address inequities in their own department might enjoy a full reading of the book, but for the general reader I recommend skimming the book to read the anecdotes, as these present the issues in a detailed yet entertaining and illuminating way.

Joint Review of[31]
Riot at the Calc Exam
and other Mathematically Bent Stories
by Colin Adams
Published by the AMS, 2009
271 pages, Softcover
Amazon prices: New $32,99m Used $42.00 (not a typo)
AMS prices: Members $26.00, Nonmembers: $32.00

and

The Great π/e Debate
Which is the Best Number?
by Colin Adams VS Thomas Garrity
Moderated by Edward Burger
Published by the MAA, 2006
DVD- 45 minutes, $24.00 at MAA, $49.00 at amazon

and

The United States of Mathematics
Presidential Debate
by Colin Adams VS Thomas Garrity
Moderated by Edward Burger
Published by the MAA, 2009
DVD- 45 minute
Amazon Price: $30.00 new, $25.00
AMS Price: non-members $30.00 new, members $25.00
Review by
William Gasarch gasarch@cs.umd.edu

1 Introduction

All three of the items can be called Mathematical Fiction (the π/e debate may be debatable). When producing a work of mathematical fiction the author had to ask himself or herself *Am I trying to educate or entertain?* I assume the author wants to do both, but in what measure? What is the right mix? They also have to ask *What math does the reader have to know ahead of time?* I will revisit these questions as I review all three items.

2 Riot at the Calc Exam

This is a collection of 33 fictional short stories where mathematics plays a role. Many of the characters in them are mathematicians. The mathematicians are *not* super genius's so we can relate to them.

Some are satires of literary genres. There are two P.I. stories (though P.I. Stands for Principle Investigator). Sample line

I was working as a snot-nosed postdoc out of a sleazy hole-in-the-wall office in L.A. Actually, UCLA to be specific.

There are two horror stories. Sample line:

"There is an easy way to learn math, and a hard way. We tried the easy way with you and it didn't work. Now we do it the hard way." He reached for her throat.

There is a medical story. Sample line:

Resident 2: *We're losing the patient*

Vinson: *Stand back! Hand me the paddles. Give me 100 volts of algebraic K-theory. Clear!*

Some place mathematics in a strange context— a fashion show

Katherine: *Mathematical Fashion has had its ups and downs over the years. Short sleeved white button down dominated the late 1960's. Is there any single item that dominates the math scene these days.*

Arturo: *Not really. Mathematicians are expressing themselves freely. They are saying I am an individual. I do sheaf cohmology and I am proud of it.*

Some of the stories have notes in the back about what some of the math is. Does the book work? As for entertainment, if you know some mathematics and like satire then this is a GREAT book (I am in this category). You do not have to know any particular math, but you have to know about math to appreciate it. Also, for some of the stories, you may need to know a bit about what they are making fun of.

The ideal audience is the same for the book *Reality conditions* that I reviewed in a prior column. Hence I quote from that column.

The ideal audience is anyone who knows and likes math at the level of an undergraduate senior or higher. For such a reader the book is a fun read. Note that it could not be used to get someone interested in math. The book will not teach you any math nor inspired you to look some up.

I didn't learn much math from reading it, though it did peak my interest to go to the web and learn a few things. However, I think the author wanted to entertain more than teach, and he succeeded.

3 The Great π/e Debate: Which is the Better Number

This is a real debate about these two numbers. It was amusing at times as the two debaters insult each other and act like real political debaters. There was even a winner in the end which I won't tell you.

Collin Adams presented the merits of π. He had some interesting things to say about π that I did not know. He also insulted both e and his opponent. Thomas Garrity presented the merits of e. He didn't really say much about it, he spend more time insulting π and his opponent. He was funnier but less informative then Adams.

It was worth seeing this debate. A math department should buy it. If it was cheaper than I might recommend an individual to buy it.

4 The United States of Mathematics Presidential Debate

This was a laugh riot and informative! The premise is that the Figure 8 Knot (Colin Adams) and the Euclidean Algorithm (Thomas Garrity) are running for *President of the United States of Mathematics* and are having a debate. Throughout the debate they satirize politics in general and have some references to the recent election (*Joe the Jacobian* is an obvious reference to *Joe the Plumber*).

The setting really works and this is really amazing! I learned some topology and some algebra from the debate. This one I would recommend for you to buy personally. Your friends interested in politics but not in math may actually enjoy it.