Review of[1]
Asymptopia
by Joel Spencer and Laura Florescu
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Reviewer: William Gasarch gasarch@cs.umd.edu

1 Introduction

This is a book on asymptotics for undergraduates; however, most of the material is not standard so a graduate student or even a professor will find something new here.

Asymptotics is, roughly speaking, what happens to a ... formula? phenomena? that depends on n when n is large. The audience for this review (SIGACT News readers) are used to this notion for algorithm analysis. There is one chapter on that topic; however, the book is mostly about asymptotics within mathematics.

2 Summary of Contents

The first five chapters are about asymptotics of factorials, big-O notation, asymptotics of integrals and sums, and asymptotics of binomial coefficients. One could view these results as interesting in their own right or as lemmas for later applications. While the distinction between lemma and application may be a matter of taste I will point out two topics that I consider applications: (1) approximations to $\int_0^1 sin^n x dx$, and (2) random walks.

The rest of the chapters are mostly applications. This is a math book so I mean applications to other fields of mathematics. We list some of the results:

1. The number of rooted trees on $\{1, \ldots, n\}$ with root 1 is n^{n-2} (this is not asymptotic nor is it proved in this book). By contrast the number of Unicycle graphs (a graph with n vertices and n edges) is asymptotically $\sqrt{\frac{\pi}{8}} n^{n-\frac{1}{2}}$. Note that the error term is additive, not (as is often the case in computer science) multiplicative.

2. There are three asymptotic lower bounds for $R(k)$ (Ramsey of k): (1) $R(k) \ge (1+o(1))\frac{k}{e\sqrt{2}}2^{k/2}$, (2) $R(k) \ge (1+o(1))\frac{k}{e}2^{k/2}$, (3) $R(k) \ge (1+o(1))\frac{k\sqrt{2}}{e}2^{k/2}$. These results are interesting and depressing. Interesting that progress has been made, and the proofs are nice. Depressing that so little progress has been made.

3. Let $\pi(n)$ be the number of primes that are $\le n$. The prime number theorem states that $\pi(n) \sim \frac{n}{\ln n}$. This is a difficult theorem. How close can we get to it just using simple combinatorics? In this chapter they show that there are constants c_1, c_2 such that

$$(c_1 + o(1))\frac{n}{\ln n} \le \pi(n) \le (c_2 + o(1))\frac{n}{\ln n}.$$

Upon seeing this my first reaction is *I bet c_1 is really small and c_2 is really large.* NO. In the proofs given $c_1 = \ln 2 \sim 0.693$ and $c_2 = 2\ln 2 \sim 1.386$. This approximation to the prime number theorem is good enough for any application in computer science.

4. If you pick three points in the unit square what is the probability that the triangle will be of size $\leq \epsilon$. This will be a function of ϵ. They show that its $\Theta(\epsilon)$.

5. There is a chapter on algorithms. This material will be familiar to most readers of this review.

6. There are two chapters on probability and there is some probability in other places. Here is one phenomena they look at from different angles: let $X_1, X_2, \ldots$ be random variables that take on the value -1 or 1, each with probability 1/2. Let $S_n = X_1 + \cdots + X_n$. Clearly $E(S_n) = 0$ but what is the probability that S_n will be far from 0?

3 Opinion

If n people read this review then with high probability $n - \ln n$ of them will find at least $\frac{2}{3} - \frac{1}{\sqrt{n}}$ of the book interesting. The proofs are readable and the results are worth knowing. One caution— some of you are used to ignoring multiplicative factors. This book is more careful about those constants so you need to get used to it. However, that is one of the benefits— it teaches you to think in a different way.

To read this book you need mathematical maturity and a basic course in combinatorics. All such people will benefit from this book since it has many results that are not that well known but perhaps should be. The triangle results above I found particularly intriguing.