Review of
Love and Math: The Heart of Hidden Reality[1]
by Edward Frenkel
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Review by William Gasarch gasarch@cs.umd.edu

1 Introduction

Mathematics has been around for thousands of years and hence has gotten the chance to become very complicated. Fields of mathematics often, over time, become connected in surprising ways. Edward Frenkel uses the analogy of different continents and bridges that are build between them. This is very apt since (1) building a bridge between (say) America and Europe would be very difficult, and (2) much of the math he talks about is very difficult.

This book is partially Edward Frankel's biography: how he got involved with mathematics, the problems he faced as a Jew in the old USSR, and his love of the subject. In order to convey this he also explains a great deal of mathematics. The mathematics is sophisticated (I will briefly present some later), yet he manages to give a sense of it. In particular he conveys that this material is interesting and important. While I was reading the book, I was also working on a fun but frankly non-important problem in mathematics. The contrast was striking!

2 Summary

The book– which is written in the first person– begins by describing some elementary particle physics and group theory and explaining how they relate. After this comes the authors own tale. Frenkel was born in 1968 in what was then the Soviet Union, and he initially wanted to study Physics. His mentor, Evgeny Evgenievich Petrov converted him to Mathematics, the subject in which he earned his PhD, but since hiw work often related to physics it is not clear he really did switch. At such high levels it is hard (and not productive) to distinguish the two.

As a young man Frenkel was a brilliant mathematician, and if the Soviet Union had not practised a form of institutionalized antisemitism, he would have passed his exams and got into Moscow University. But under the circumstances such sucess was impossible no matter how well he performed. The examiners kept making the questions harder and harder, both in terms of intellectual merit (which he as still able to solve) and in terms of stupid pedantics. As an example, he gave the definition of a circle as 'the set of points equidistant from a point' which was deemed wrong since it should be 'the set of all points equidistant from a point'. Another way for Jews to be barred from school was to give them harder problems to solve. Often they had an easy solution that was hard to find, thus giving the appearence of fairness. For more on this see *Jewish Problems* by Khovanova and Radul, here: `http://arxiv.org/abs/1110.1556`.

Frenkel's experience was far from unique, and in this book he describes the ways in which Jewish Mathematicians, Physicists, and other scientists dealt with this system. Some of them met quasi-secretly and still managed to get much done. It is tempting to wonders if this oppression got their creative juices flowing; however, this is a fallacy. We only read about those who managed to do well. I am sure that many brilliant students were blocked from making contributions. Their biographies are not written.

I am curious about what caused this antisemitism and what its effect (surely negative) was on the Soviet Union. The book does not go into this since it is his story and does not pretend to be a sociology or history book. However, it is good to have his story documented.

At the heart of the book is the Langlands program which is an attempt to build those *bridges* between different mathematical *continents*. Here is an example. Let $f(x, y)$ be a cubic polynomial with integer coefficients. If p is a prime we can ask how many $(a, b) \in Z_p \times Z_p$ such that $f(a, b) = 0$. we denote this n_p. We can then form an infinite polynomial $g(x) = \sum_{p \text{ prime}} n_p x^p$. This infinite polynomial is then associated to a group G of symmetries in the complex plane called a modular form. This correspondence is 1-1 and preserves some properties. That is, every cubic polynomial maps to a modular form and every modular form maps to a cubic equation. This is an important connection.

What the Langlands program does is essentially take the notion of *polynomial* and generalize it, and take the notion of *modular form* and generalize that. The program then makes conjectures about how these very abstract objects relate. Frenkel illustrates this connection-building process with several nice examples until, on page 222, he has a chart that connects Number Theory, Riemann Surfaces (Geometry), and Quantum Physics. Quantum

Physics? How did that get in there? Through Gauge Theory– a complicated notion that Frenkel (wisely) does not try to explain. However, having read this book I now want to find out.

Frenkel claims that frequently a branch of math that was thought of as pure abstraction ends up being applied to practical problems. I am often skeptical of such claims since the (perhaps forgotten) origin of a field is often some real world application. However, the examples given here seem legitimate since Number Theory really has no apparent connection to Quantum Physics yet the links are there. One is left with the impression that Frenkel and others in the book (including Ed Witten, the only Physicist to win a Field's Medal) are serious brilliant people who are doing serious brilliant work.

3 Opinion

You do not need to know much math to read this book, but you need to like it. Depending on your level you will get lost at some point (I got lost on the definition of a *Sheaf*). However, this is not a book to read to learn math. Its a book to read to be inspired to learn math.