Joint Review of[1]

Once Upon a Prime:
The Wondrous Connections Between
Mathematics and Literature
by Sarah Hart
Publisher: Flatiron Books
$14.99 Kindle, $18.99 paperback, $22.74 Hardcover
290 pages. Year: 2023

and

Arm in Arm
by Remy Charlip
Publisher: NYR Children's Center
$19.95 hardcover, Much cheaper used. 40 pages
Year: Originally 1969. But Also 2019

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1 Introduction

In 1970 I read *Arm in Arm*. It was a delightful book (for a 9 year old) full of word play and logic. Either I liked the book because I liked math or I liked math because I liked the book. The cliche comment is to ask *which came first, the chicken or the egg?*. Two comments on that:

1. This cliche is no longer accurate since *the chicken came first* (see link here). In any case, I like math and I like the book.

2. The book *Arm in Arm* has a picture of a Chicken and and Egg with the following conversation.

   Chicken: Who was first, me or you?

   Egg: Don't question it, be grateful we have one another.

In 2023 I read *Once Upon a Prime: The Wondrous Connections Between Mathematics and Literature*. To briefly describe it I can't to better than the title itself.

Why review these books together? *Once Upon a Prime* talks about, say, how Hamlet has narrative distance since there is a play-within-a-play. *Arm and Arm* uses recursion which is similar. More generally, *Arm and Arm* is an example of some of the points the book makes. Note that *Arm and Arm* is a 40-page illustrated books for children (recommended for ages 6–9 though I think it should be 6-12 plus a 60 year old reliving his youth); by contrast, *Once upon a Prime* is about serious literature.

2 Once Upon a Prime

2.1 Summary of Contents

The book has three Parts. Part I has 4 chapters, Part II has 3 chapters, and Part III has 3 chapters.

For each chapter I discuss a few points it makes. Each chapter makes far more points then I discuss.

Part I: Mathematical Structure, Creativity, and Constraint

Chapter 1 (titled *One, Two, Buckle My Shoe: The Patterns of Poetry*) considers poetry.

1. How would you describe a limerick? The rhyming pattern is AABBA. This chapter gives a delightful example of a way to generate 3^5 limericks by having a choice of 3 for each line. All 3^5 limericks make sense!

2. Some Sanskrit poetry has a limit on the length of each line. But what is length? Some syllables are light and count 1 for length, and some are heavy and count 2 for length. How many patterns for n-length lines are there? The reader can work out that its the nth Fibonacci number. This is not so much an *application* of Fibonacci numbers as one of the *origins* of Fibonacci numbers, which predates Fibonacci by a few hundred years, or a thousand years, depending on who you count. This is briefly discussed. A while back I did a blog post on the origins of Fibonacci numbers (see link here).

Neither of the point above are about the *content* of the poem. We present two poems where the content is mathematical.

From Chapter 1:

As I was going to St. Ives
I met a man with seven wives
Each wife had seven sacks
Each sack had seven cats
Each cat has seven kits
Kits, cats, sacks, and wives
How many were going to St. Ives?

I leave it to the reader to solve it. Hint: you can do it without any calculation.

The following poem was not in the book, but I include it since I like it:

A challenge for many long ages
Had baffled the savants and sages
  Yet at last came the light
  Seems that Fermat was right
To the margin add 200 pages

(I actually do not know who first wrote this. If you do, please email me.)

Chapter 2 (titled *The Geometry of Narrative: How Mathematics can structure a story*) discuses how a story can have a mathematical structure.

1. A story can have parts that are happy and sad. Think of the classic *Boy meets Girl*, *Boy loses Girl*, *Boy regains Girl*. That can be thought of as a function that goes up then down again and then up again. There are more complicated structures as well.

2. The book *The Luminaries* by Eleanor Catton has 12 chapters. For $1 \leq i \leq 12$, chapter i has $13 - i$ sections. Hence, for all $1 \leq i \leq 12$,

   Chapter Number + Number of Sections = 13.

   Also, each chapter is half the size of the previous one. The book has some interesting numbers like 4096 ($4096 = 2^{12}$). The book won the Booker prize.

The Luminaries raises the following point: does using math to structure a book make it a better book? For *The Luminaries* the structure and the plot and the characters are interrelated so the answer is YES, and the fact that the book won the Booker prize is very strong evidence that it's a great book.

Chapter 3 (titled *A Workshop for Potential Literature: Mathematics and the Oulipo*) begins talking about a group (or perhaps a set) of people who, in 1960, formed *Ourvoir de Littrauture Potentielle*, or *Oulipo* for short. This translates roughly to *Workshop for Potential Literature*. Their goal was to invent new structures for literature. This chapter then looks at ways to structure literature, some of which came from this group.

One way is to put constraints on what one writes. *La Dispartition (in French) and A Void (in English)* are a French book, and its English translation. Neither one uses the letter e (translating *La Dispartition*, from a French book with no e's to A Void, an English book with no e's must have been really hard). *Gadsby* is a book (in English) which does not use the letter e. This chapter describes why, for *La Dispartition*, this constraint added to the book (hence it was not just a gimmick); however, for *Gadsby* it added nothing (hence it was just a gimmick). Far more complicated structures and constraints, for other books, are discussed. One uses the Fano plane. So there is some serious math here.

While reading this chapter I thought they have it backwards: rather than come up with a structure (perhaps a gimmick) to write a book (e.g., the ith chapter has i^2 sections), first write your book and then see how you can structure it better to reflect the plot or characters. This chapter does point out that sometimes these structures are just a gimmick.

Chapter 4 is titled *Let me Count the Ways: The Arithmetic of Narrative Choice*. True Story: I was at the Worlds Fair in Montreal in 1968. One of the exhibits was that you watch a movie, but at certain points in the movie the audience voted on how a crucial plot point would go. I was annoyed since my vote usually lost and so I never did found out how my choices would have worked out. There are works of literature that use the same principle. This chapter discusses those and other mechanisms where a story may actually be several stories.

Part II: Algebraic Allusions: The Narrative Uses of Mathematics

Chapter 5 (titled *Fairy-Tale Figures: The Symbolism of Numbers in Fiction*) explores the following question: Why do some numbers occur in literature more than others. Note: 3 (3 wise men, 3 wishes, Goldilocks and the 3 bears, 3 witches in Macbeth, the Trinity), 7 (7 Dwarfs in Snow White, the 7th seal, 7 deadly sins), 12 (12 tribes of Israel, 12 Apostles), 40 (Ali Baba and the 40 thieves). There are also large numbers: 100, 1000, and sometimes 99 or 999, that appear a lot. This chapter gives intelligent speculation about why these numbers occur so often. This is not a scholarly study; however, I am glad about that. A scholarly study would be more boring and just as speculative.

Chapter 6 (titled *Ahab's Arithmetic: Mathematical Metaphors in Fiction*) points out that many works of literature have actual math in them if you know to look for it. *Moby Dick* by Melville

actually talks about cycloids (This chapter gives strong evidence that Melville knew some math.) *Daniel Deronda* by George Eliot talks about statistics. There are many other examples given. None of these books have mathematicians or scientists as characters.

Chapter 7 (titled *Travels in Fabulous Realms: The Math of Myth*) *uses* math and science to determine if facts about mythical creatures (e.g., the Lilliputians (6 inches tall), Brobdingnag (72 feet tall), various space aliens) can be true. Largely the answer is NO. The chapter also looks at the size and shape of real animals to determine if they could be larger or smaller.

Part III: Mathematics Becomes the Story

In Chapter 6 it was noted that some works of literature *use* mathematics. However, none of the books discussed were *about* mathematicians or math. Chapter 8 (titled *Taking an Idea for a Walk: Mathematical Concepts so Compelling they escape into Fiction*) discusses books where math plays a central role. Often the math is used since it was somehow in the public mind at the time. These include the following.

1. *Flatland* by Edwin Abbott: This book is about a society which is two dimensional. It is a satire of Victorian England.

2. *The Time Machine* by H.G. Wells: Time is the fourth dimension and the machine is able to travel in that dimension.

3. *Jurassic Park* by Michael Crichton: DNA from the past is used to create Dinosaurs. Chaos theory and fractals are used and are an important part of the story.

These books are about serious math, and this chapter explains this serious math very well. Even though the readers of this column are mathematically sophisticated, they might learn some math from this chapter that they didn't already know.

Chapter 9 (titled *The Real Life of Pi: Thematic Mathematics in the Novel*) discusses how math can be an underlying theme in some books. The constant π (actually approximations to it) is used in *Life of Pi*. Then it goes on to discuss the book *The Library* which leads to some intricate combinatorics and geometry. Then Lewis Caroll's *Alice in Wonderland* and *Through the Looking Glass* are discussed (note that Lewis Caroll was a mathematician) since they use some math and physics. Finally they discuss *The Hitchhiker's Guide to the Galaxy*.

Chapter 10 (titled *Moriarity was a Mathematician: The Role of the Mathematical Genius in Literature*) considers how mathematicians are portrayed in literature. They are often genius's. They are often unemotional, which is not accurate. This chapter gives pointers to lots of books that portray mathematicians, some quite well.

Who should read this book?

For a mathematician who wants to know how literature both used math and can be analyzed by math, this is a great book. There is even some math it of interest that a mathematician might not know. After reading it, a mathematician may want to read some of the books mentioned.

What if someone does not know any math? The book would be rough in spots. After reading it, a non-mathematician may want to look up some math.

3 Arm in Arm

Arm and Arm has lots of cute poems and very short stories and ... hard to say what it really is. I

gave one example in the introduction (the Chicken and Egg conversation). I'll give one more. There are two pictures pictures on two pages that face each other. Note that the book is illustrated, so unless SIGACT News wants to pay for color printing and an artist, I must make due with telling you about the picture, and giving you the words.

1. One of the pictures is mostly snow and 6 children. There is also a window, presumably of a house. One of the children says the following: *Isn't it better to be out in the cold snow saying "isn't it better to be out in the cold snow rather than in a warm bed" rather than in a warm bed saying "isn't it better to be out in the cold snow rather than in a warm bed".*

2. The other picture is of a large bed with 6 children. There is a window which indicates that it is snowing outside. One of the children says *Isn't it better to be in a warm bed saying "isn't it better to be in a warm bed rather than out in the cold snow" rather than out in the cold snow saying "isn't it better to be in a warm bed rather than out in the cold snow".*

Who should read this book?
This is a great gift for anyone from 6 to 12 years old.

4 Coda

Once Upon a Prime does not discuss the following:

1. The book *Reality Conditions* by Alex Kasman, which is a collection of short stories by a mathematician, about math and mathematicians. I reviewed it (see link here).

2. The book *Riot in the Calc Exam and other Mathematically Bent Stories* by Colin Adams. which is a another collection of short stories by a mathematician, about math and mathematicians. I reviewed it (see link here).

3. Movies and TV show that have a math theme (e.g., the movie *Proof* in a big way, the movie *Mean Girls* in a small way, and the TV show *Numb3rs* in a big way). I reviewed *Numb3rs* (see link here). Note that Math in Movies and TV shows would be another book.

The above noted omissions are *not* a criticism. Au contraire, the fact that I would like to see books about the math in these venues (perhaps by Sarah Hart) shows that, like a good novelist, she leaves us wanting more.

Arm in Arm is short at 40 pages. That I want more is a compliment to the author and illustrator Remy Charlip.