Referee's Report for
On the two-colour disjunctive Rado Number for the equations $\sum_{i=1}^{m-2} x_i + ax_{m-1} - x_m = c - j,\ j = 1, 2$
Author of Paper: Dwivedi and Tripathi

1 Final Decision

2 Abstract

Your $\mathcal{R}$ has a subscript of d that is not used.

3 Introduction

1. Page 1. There is a lot missing in your discussion of Rado's theorem which sets the stage for your work. In particular there are two kinds of questions to ask and you don't distinguish them. I will discuss this for the case of one equation.

    - **Rado's Theorem for Single Equations:** Let $a_1, \ldots, a_m \in \mathsf{Z}$ and let $E(x_1, \ldots, x_m) = \sum_{i=1}^{m} a_i x_i$. the following are equivalent

      (a) for all r there exists n such that for all r-colorings of $\{1, \ldots, n\}$ there is a monochromatic solution to E. (Note: the value of n from the standard proof is enormous and the smallest n that works is thought to be much smaller.)

      (b) some non-empty subset of $\{a_1, \ldots, a_m\}$ sums to 0.

    - Say b above is false. Let M be the max sum of all nonempty subsets of $\{a_1, \ldots, a_m\}$. Let p be the least primes bigger than M. There is an $M - 1$-coloring of N with no mono solution. But what about smaller values like $M - 2$? Or as in this paper, 2. So the question is:

      for equations that do not satisfy b, what happens with 2-coloring?

    - Another variant of Rado's theorem is to allow a non-zero constant on the right hand side.

    - Your paper looks at both variant at the same time: no sum is zero, and the RHS is not zero.

2. Page 2. You do not say what the subscript d means so I assume it means disjunctive. This is not needed. Stop using the d subscript since it makes your reader look for the parameter d.

3. Page 2 and Theorem 3. I can't see where $\mathcal{R}(c)$ is. The definitions of $\mathcal{R}$ should be in definition environment so are easier to find.

4. Page 2. Were Schaal & Zinter the first paper to consider the case where a constant (c) was allowed?

4 Results for $\sum_{i=1}^{m-2} x_i + ax_{m-1} - x_m = c_i$, $i = 1, 2$

1. The title of this section of the report is exactly the same as the title of section 2 in the paper. Note the very confusing typo: you use i in both $\sum_{i=1}^{m-2}$ and in c_i. Replace

   $\ldots = c_i$, $i = 1, 2$

   with

   $\ldots = c_j$, $j = 1, 2$

   I talk more about this later.

2. Page 3. Proposition 4. You begin with $(1, \ldots, a)$ but never use that. Replace the first two sentences with

   Let $a, \lambda, n \in \mathsf{N}$ such that $a \geq 3$, $n \geq 1$ and $\lambda \geq a - 1$.

3. Page 3. Line -2.*soloution* should be *solution.*

4. Page 3. Theorem 5 is far less interesting than it appears.

   - The coloring of $[1, k-1]$ that has no monochromatic solution has no solution at all.
   - The proof that any 2-coloring of $[1, k]$ has a monochromatic solution is $x_1 = \cdots = x_m = k$. So the x_i's are all the same color since they are all the same number.

   I am *not* suggesting you remove Theorem 5. I suggest that you propose (or solve) open questions that ask for a more interesting solution. For example

Open Question: Investigate a variant of $\mathcal{R}$ where there is an condition that the monochromatic solution can't be all the same number.

5. The notation is inconsistent and confusing. Note the following.

   - The title of Section 2 is equation we are dealing with is

     $$\text{Result for } \sum_{i=1}^{m-2} x_i + ax_{m-1} - x_m = c_i$$

     You used the index i twice and in different ways. That is the i in $\sum_{i=1}^{m-2}$ and the i in c_i are different.

   - Proposition 4 uses $\sum_{k=1}^{n} x_k + ax_{n+1} = N$.

     You should not use k as an index since later k is used in $c = k(a + m - 3)$.

   - Theorem 5. The statement uses c_i with $i = 1, 2$. Do you also use $\sum_{i=1}^{m-2}$? Implicitly since you discuss $\mathcal{R}(k_1, k_2)$. And you *do* use that sum in the proof itself. So again you are using i two ways.

6. Page 3. Equations 2a and 2b use c_1', c_2', a. After the equation you say what x_1', c_2, a' are. But a' was never used in Equations 2a and 2b. I think you meant to have a' instead of a in Equations 2a and 2b. Please check before making the change.

7. Theorem 6 is hard to read (though see next point.) You should have before Theorem 6 a statement and proof of an actual example of the theorem.

8. Theorem 6. c_j' is defined but only used in the upper bound. Yous should state Theorem 6 in a more readable way without c_j' or a' and then rephrase it that way only when needed.

9. Theorem 6. Lower Bounds. Readability. You need to have titles in boldface or italics to separate the cases like this:

   Cases 1 and 4

   Cases 2 and 3

10. Theorem 6 ends up with the monochromatic solution having many of the variables be the same. State as an open problem obtaining a monochromatic solution with all of the variables different. Or at least more of them different.

5 Open Problems Section (You should have one)

1. Theorems 5 and 6 cover many but not all cases of the equation

$$\text{Result for } \sum_{i=1}^{m-2} x_i + a x_{m-1} - x_m = c_i$$

   You should have an open problems section where you state

   - The simplest case that is open.
   - The set of cases that is open (if that is easy to state).
   - Your opinion if you have one.

2. Your results are about linear equations with coefficients all 1's except for one -1 and one a. What about other linear equations?

3. Your results are all about 2 colors. What about 3 or more?