Supplemental Exercises: Unit 5

Scientific Computing with Case Studies

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SIAM Press, 2009

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1. Suppose we have used a PECE algorithm with the formulas:

$$
\begin{aligned}
y_{n+1} = &\quad y_n + \tfrac{h}{12}(23f_n - 16f_{n-1} + 5f_{n-2}) \qquad \mathit{error}: \frac{3h^4}{8}y^{(4)}(\xi) \\
y_{n+1} = &\quad y_n + \tfrac{h}{12}(5f_{n+1} + 8f_n - f_{n-1}) \qquad \mathit{error}: -\frac{h^4}{24}y^{(4)}(\eta)
\end{aligned}
$$

Assuming that f_{n-2}, f_{n-1}, and f_n are correct, give a computable estimate of the local error in using the predictor as an approximation to the true solution.

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2. Write MATLAB code to estimate $\boldsymbol{y}(1)$ using Euler's method with stepsize $h = 0.1$, given

$$
\begin{aligned}
\boldsymbol{y}'(t) &= \begin{bmatrix} 2ty_{(1)}(t) + y_{(2)}^2(t) \\ y_{(1)}(t)\cos(y_{(2)}(t)) \end{bmatrix}, \\
\boldsymbol{y}(0) &= \begin{bmatrix} 1 \\ 2 \end{bmatrix}.
\end{aligned}
$$

Notation:

$$
\boldsymbol{y}(t) = \begin{bmatrix} y_{(1)}(t) \\ y_{(2)}(t) \end{bmatrix}.
$$

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3. Consider the DAE from Chapter 21 for modeling the spread of an infection:

$$
\begin{aligned}
\frac{dI(t)}{dt} &= \tau I(t)S(t) - I(t)/k \\
\frac{dS(t)}{dt} &= -\tau I(t)S(t)\,, \\
1 &= I(t) + S(t) + R(t).
\end{aligned}
$$

We are given values for τ and for $I(0)$, $S(0)$, and $R(0)$.

(a) Without modifying the equations by differentiation or substitution, write this system in the form $\boldsymbol{M}\boldsymbol{y}' = \boldsymbol{f}(t, \boldsymbol{y})$, where $\boldsymbol{M}$ is a 3×3 matrix.

(b) If $\boldsymbol{M}$ is nonsingular, then `ode23s` should be used to solve this problem. Otherwise, `ode15s` should be used. Which of these two algorithms would you choose?

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4. Let

$$u'' = \cos(t)u'(t) + \sin(t)u(t),$$

with $u(0) = 0$ and $u(1) = 1$. Let $h = 1/8$.

(a) Write a set of finite difference equations that approximate the solution to this problem at $t = jh$, $j = 0, \ldots, 8$.

(b) Write these finite difference equations in the form $\boldsymbol{A}\boldsymbol{u} = \boldsymbol{b}$, where $\boldsymbol{A}$ is a matrix and u_j is your approximation to $u(jh)$.
