

The Phantom of the Opera_{tions}

Replacement for the pointer

Stokes Theorem The Stokes Theorem (also called the Green's Theorem, or, in 3-D, the Divergence Theorem) relates an integral over the interior of a region to an integral over the boundary of the region. We can use it to estimate the area of the phantom. The resulting formula for the area is

$$\frac{1}{2} \int_0^{2\pi} \mathbf{z}(\theta)^T \mathbf{x}(\theta) d\theta,$$

where $\mathbf{x}(\theta)$ is a point on the boundary and

$$\mathbf{z}(\theta) = \begin{bmatrix} \frac{dx_2(\theta)}{d\theta} \\ -\frac{dx_1(\theta)}{d\theta} \end{bmatrix}.$$

In other words, the area is

$$\frac{1}{2} \int_0^{2\pi} \left[x_1(\theta) \frac{dx_2(\theta)}{d\theta} - x_2(\theta) \frac{dx_1(\theta)}{d\theta} \right] d\theta.$$