Fall 2007 **CMSC 858K: Homework 2** Samir Khuller

Due in class: Sep 18.

Problems are to be done by yourself (unlesss specified) with no help from another person. If you use any sources other than the textbooks for the class, you are to cite them. In any case, the writeup should be your own and not simply taken from another source.

(1) A *maximal matching* is a matching M such that no edges can be added to the matching, without destroying the property that it is a matching. In other words, for any edge e, $e \cup M$ is not a matching. Give a linear time algorithm for finding a maximal matching in a graph. Then prove that the size of the maximal matching is at least half the size of a maximum matching.

(2) Consider a set of costs $c_{ij} \geq 0$ on the edges of a complete graph K_n with n nodes, and let $S \subseteq \{v_1, \ldots, v_n\}$. We wish to find a subgraph of K_n that has odd degree at nodes in S, an even degree (possibly zero) at all other nodes, and as low cost as possible. Show that this is a minimum weight matching problem. (Assume that min weight matching can be solved in polynomial time in graphs that are not necessarily bipartite.)

(3) We are given a connected graph $G = (V, E)$ and a cost function $c : E \rightarrow Z^+$. We wish to find a walk, traversing each edge of E at least once, such that the total cost of the walk (with multiple traversals of an edge charged multiply) is as small as possible. Give a polynomial time algorithm for this problem. (You may assume a solution to problem (2) even if you did not solve problem (2).)

(4) Design a polynomial time algorithm for the following problem. You have a directed network, with two special vertices s and t. Each edge e has a positive capacity $c(e)$ and a positive delay $d(e)$. The time to ship σ units of data from s to t on path P is $\sum_{e \in P} d(e) + \frac{\sigma}{C}$ where $C = \min_{e \in P} c(e)$. Design a polynomial time algorithm to find the path that will minimize the shipping time? You should also give an argument that shows why your method works.

(5) Let $G = (X, Y, E)$ be a bipartite graph. Assume for simplicity that $|X| = |Y|$.

A subset $S \subseteq (X \cup Y)$ is called a *vertex cover* if for *each* edge $e = (x, y)$ in E either $x \in S$ or $y \in S$ (or both).

Let M^* be a maximum matching in G.

(a) Prove that if S is a vertex cover then $|S| \geq |M^*|$.

(b) Give a polynomial time algorithm to find the minimum cardinality vertex cover. Give a proof for its correctness.