Fall 2007 **CMSC 858K: Homework 5** Samir Khuller

Due in class: Oct 30.

(1) Give an f-approximate algorithm for the set cover problem in which each element belongs to at most f sets.

(2) Prove that Integer Programming is NP-hard.

(3) Jerry runs a tent rental company. People make requests for reserving tents. Request i has three parameters - the profit p_i gained if the request is satisfied, the start time s_i and the end time e_i. Assume that n requests are known in advance. Jerry would like to maximize his profit.

  (a) Design an efficient algorithm for the case when Jerry has *one* tent, and would like to maximize his profit. Note that if Jerry agrees to rent the tent to a set of people, then the time intervals during which they wish to have the tent should not overlap. (No proof required.) Faster algorithms will receive more points.

  (b) If Jerry has k tents, how would you reduce this problem to an instance of min-cost flow? (No proof required.)

(4) Consider the following approximation algorithm for unweighted vertex cover. Run DFS in the graph (assume it is connected) to obtain a DFS tree T. Now pick all the non-leaf vertices in the vertex cover. Prove that this is a 2-approximation. Why does this form a vertex cover? (Hint: Any matching in G is a lower bound on the size of a vertex cover.)

(5) Show that there is always an optimal solution to the vertex cover LP whose optimal solutions only have the values $\{0, \frac{1}{2}, 1\}$. (If you look up the proof, I would like you to describe it in your own words.)