Fall 2007 **CMSC 858K: Homework 6** Samir Khuller

Due in class: Nov 13.

(1) Design a constant factor approximation algorithm for the weighted $K$-Center problem, which is defined as follows. (This tries to capture the fact that not all locations are equally important, in other words, we may want a center closer to important locations by giving them a higher weight.) The weighted generalization is formally defined as follows: given a graph $G$ and a weight function $w(v)$ defined on $V$, find a subset $\mathcal{S}$ of vertices of size at most $K$ such that each vertex is "close" to a vertex in $\mathcal{S}$, where the distance from vertex $u \in S$ to vertex $v$ depends on the weight of $v$. Formally, we change the definition of the cost measure to

$$\min_{S \subset V, |S| \leq K} \max_{v} d(v, S) \cdot w(v)$$

where $d$ is the distance function. We define $d(v, S)$ as $\min_{u \in S} d(v, u)$.

(2) Page 23, exercise 2.6 (vertex coloring).

(3) Consider an instance of the Set Cover problem over the set of elements $X = \{x_1, x_2, \ldots x_n\}$. Let $\mathcal{S} = \{S_1, S_2, \ldots, S_m\}$ be the collection of sets. We are also told that **each** element $x_i$ belongs to *at least* $\frac{m}{2}$ sets in $\mathcal{S}$. Suppose we run the greedy algorithm to find a "small" set-cover (i.e., we pick a set that is covering the largest number of uncovered elements). Prove that the number of sets picked is $O(\log n)$.

(4) Page 25, exercise 2.15 (maximum coverage).

(5) Design a 1.5 approximation algorithm for the start-anywhere end-anywhere Traveling Salesman Path problem in a complete graph where the edge weights satisfy triangle inequality. Design an algorithm based on Christofides method.