Fall 2007 **CMSC 858K: Homework 8** Samir Khuller

Due in class: Dec 4.

(1) Problem 5.2 (page 52).

(2) Problem 17.1 (page 144).

(3) In class we studied the vertex cover problem in which our goal was to cover *all* the edges using minimum weighted set of vertices. Consider the problem of covering a *fixed* number of the edges instead of all the edges. Let the parameter p which is part of the input denote the number of edges to be covered. The integer program for this problem is given below. The LP can be obtained by relaxing the integrality constraints.

$$\begin{aligned} \min \quad & \sum_{j=1}^{n} w_j \cdot x_j \\ \text{subject to} \quad & \\ & x_i + x_j + y_{ij} \geq 1, \forall (v_i, v_j) \in E \\ & - \sum_{(v_i,v_j)\in E} y_{ij} \geq -(n-p) \\ & x_j \in \{0,1\}, j = 1, 2, \ldots, n \\ & y_{ij} \in \{0,1\}, \forall (v_i, v_j) \in E \end{aligned}$$

Design a primal dual approximation algorithm for this problem with a constant approximation factor.

(4) Write an IP (Integer Program) formulation for the Steiner Tree problem. Write the relaxation of this IP to the LP. Write out the dual of this LP.

(5) Give an example showing that there is a "gap" between the optimal integral solution for the Facility location IP, and the corresponding LP relaxation. Prove that this gap is some constant $c > 1$. In other words, argue that the optimal integral solution has cost more than the fractional solution by a factor of at least $c > 1$.