Penguins on Ice: Solution

Problem: You are given an **integer grid** [0..size]x[0..size], and a collection of **circles**, each given by its center (cx, cy) and radius r. Find the **shortest path** from (0,0) to (size,size) that avoids all the circles and makes **at most one turn** (at a vertex).

Solution Strategy:

- Try **all possible turning points** (x,y), where $0 \le x,y \le$ size (using two nested for loops), and for each check that the path is **valid**, that is, neither of the line segments:

  (0,0) to (x,y) and (x,y) to (size,size)

  hits any circle.
- **If valid**, then compute its **length** as:

  sqrt(x^2 + y^2) + sqrt($(size-x)^2$ + $(size-y)^2$)
- **If no path is valid**, return (-1,-1). Otherwise, return the turning point that gives the **minimum length**.

Hitting an Ice Berg?

Key Utility:

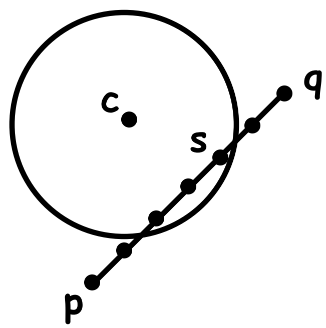
- Given a line segment from **p**=(p_x,p_y) to **q**=(q_x,q_y), does it hit circle with center **c**=(c_x,c_y) and radius r?

Quick-and-dirty (and wrong) Solution:

- Break the line segment up into **many small points**. For each point, test whether it **lies within the circle**. If any do, then declare the segment to be invalid.
- **Problem**: Did you pick enough points?
- Let m be the number of pieces, say m = 100.
- For i = 0 to m, let $\alpha = i/m$ and let $s = (1-\alpha)\cdot p + \alpha q$:

  $s_x = (1-\alpha)\cdot p_x + \alpha q_x$ and $s_y = (1-\alpha)\cdot p_y + \alpha q_y$
- Test whether $\text{dist}(s, c) \leq r$, where

  $$\text{dist}(s, c) = \text{sqrt}((sx-cx)^2 + (sy-cy)^2).$$



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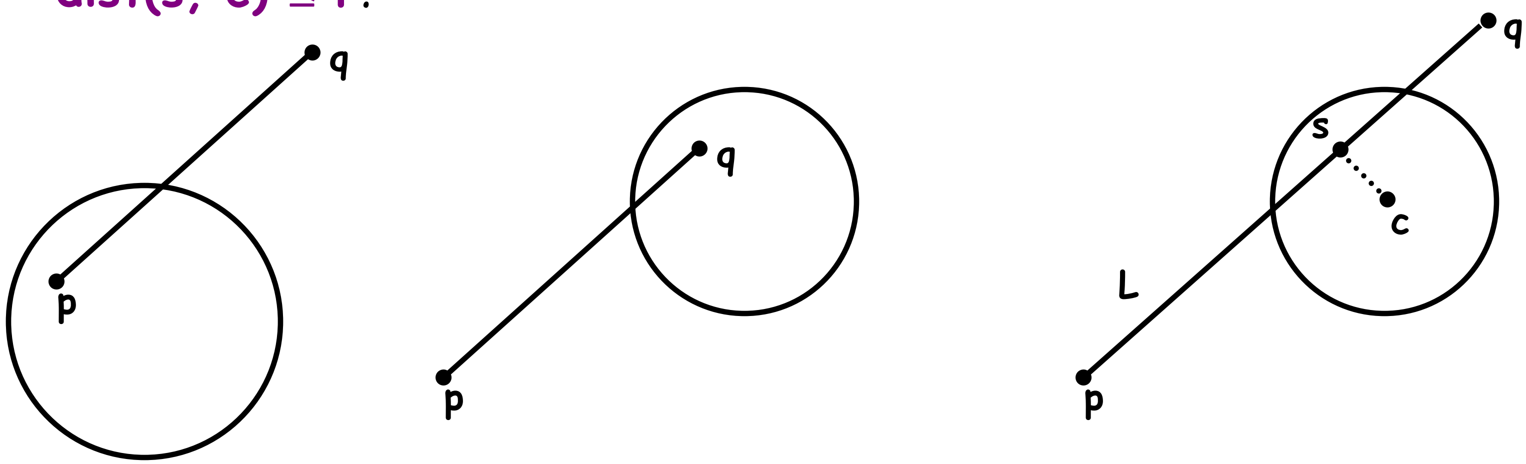
Correct Solution: There are three cases.

- Does p lie within the circle? dist(**p**, **c**) $\leq$ r?
- Does q lie within the circle? dist(**q**, **c**) $\leq$ r?
- Let L be the (infinite) line passing through **p** and **q**. Let s be the closest point on this line to **c**. Does **s lie between p and q** and is **dist(s, c) $\leq$ r**?



Hitting an Ice Berg?

Computing the closest point to L: There are many solutions. This one is based on vectors.

- Let **u = c − p** be the vector from **p** to the circle center.
- Let **v = q − p** be the vector from **p** to **q**.
- Let

$$\alpha = \frac{u_x v_x - u_y v_y}{v_x^2 - v_y^2}$$

- If α **< 0, then p is closest** to center. If α **> 1, then q is closest** to center, otherwise, let **s = (1−α)·p + αq**.