A Knight and A Queen
Problem

Given a Knight and a Queen on a chessboard, find if the Knight can reach the Queen within a bounded number of moves $k$. 

![Chessboard with Knight and Queen](image-url)
First Idea

Breadth First Search (BFS)
Positions and Levels

- Represent positions on the board as triples \((x, y, l)\)
- \((x, y)\) represents the coordinates of the position on the board
- \(l\) represents the minimum number of moves from the starting position to reach the current position
Algorithm

- Put the position of the Knight in a queue as well as in a set $S$ with level 0
- While the queue is not empty
  - Pop a position $p$ from the front of the queue. If its level is $k$ ignore it and continue to the next iteration.
  - Consider the 8 neighbor positions the Knight can go to from $p$
  - Add those that are on the board and not in $S$ to the queue with level one more than the level of $p$
Algorithm

- At the end, $S$ contains all positions reachable from the Knight’s start position within $k$ moves
- Check if Queen’s position is in the set $S$ (note that the set ignores levels of positions)
Input Size

- Pay attention to the bounds of the size of the input in the problem statement.
- The board can be as big as 1,000,000 x 1,000,000.
- You cannot use a matrix to keep track of visited positions during the BFS.
Input Size

- The number of moves can be as big as 256
- How large can $S$ get? A position can have up to 8 children!
- Try it! Print the size of $S$ at the end of the algorithm when the start position is sufficiently far from the edges of the board
- $915973$!
- Times out!
A Better Idea

- Generate reachable positions from both Knight’s and Queen’s start positions going as long as $\frac{k}{2}$ moves into sets $S_1$ and $S_2$.

- Check if their intersection is nonempty. If yes, the Queen is reachable by the night within $k$ moves.

- When $k = 256$, $S_1$ and $S_2$ have at most $228613$ elements. Better!