Transition Logic

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Idea of Logic

- Knowledge state: $KB \land K_0 \models_{cog} K_i$
- $\models_{cog}$ formally describable by $\models$
- $\models$ studied for 2000+ years
- relationship of $\models_{cog}$ and $\models$ studied scientifically only for a few decades in CogSci (before through introspection and commonsense psychology)
Dimensions of Logic Space

- Structural explorations in a given logic
  - theorem proving, proof theory, semantics
- Problems of representation in a logic
  - eg. how to represent change in FOL
- Design of a logic for modelling cognitive phenomena
  - find $\models$ modelling cognitive logical knowledge relations (eg. including change)
Knowledge Processing

- . . . for agent in an environment
- Representation for fast retrieval
- Exploiting logical relations
  - static aspect for knowledge compression
- Dynamics of knowledge state
  - dynamic aspect (need for compression)
- Both aspects fundamental for reasoning
  - eg. in planning, explanation, diagnosis etc.
What Is a Logic?

- A formal language for representation +
- a relationship $\vdash: KB \land K_0 \vdash K_i$
- Enables reduction of KB (compression)
- But what if KB changes to KB'?
- Natural reasoning suggests:
  - " $KB \land K_0 \land t_0 \vdash K_i$"
- Our solution: $KB \land K_0 \vdash_{t_0} K_i$
  - solves compression and frame problem
Natural Reasoning
Under Changes

- Example: purse with credit card in pants pocket; put on pants at home; go to restaurant: credit card with me?

- Formally *static*: \( \text{in}(cc, purse); \text{in}(purse, pants); \text{loc}(me, at home); \text{loc}(pants, at home); \text{in}(x, y) \land \text{loc}(y, z) \rightarrow \text{loc}(x, z); \)

  *transitions*: \( \text{loc}(me, x) \land \text{loc}(pants, x) \Rightarrow \text{loc}(me, x) \land \text{loc}(pants, x) \land \text{in}(pants, me); \)

  \( \text{loc}(me, at home) \Rightarrow \text{loc}(me, rest); \)

  \( \text{loc}(cc, rest)? \)
A Simple Didactic Example $E_1$

- $U, U \rightarrow F \vdash F / H, H \rightarrow A \vdash A / D, D \rightarrow O \vdash O$
  - $U$: block is uncovered
  - $F$: block is free
  - $H$: hand holds block
  - $A$: block is airborne
  - $D$: block is put down
  - $O$: block is on table
- $U, U \rightarrow F$ is short for $U \land (U \rightarrow F)$
Transitional Problems

- A transition is a pair \((E,F)\) of conjunctions of atoms
- A transitional problem \((W,T,I,G)\)
  - \(W\): world knowledge (definite nonrec. cls.)
  - \(T\): set of transitions
  - \(I\): initial state (set of atoms)
  - \(G\): goal state (set of atoms)
  (finite sets; set, conjunction interchangeable)
Source informally

- $\sigma(F) = \{\{U\}\}$ in example
- for $U_1 \land U_2 \rightarrow F$, $\sigma(F) = \{\{U_1\},\{U_2\}\}$
  - $U_1$ musty, $U_2$ lack of oxygen, $F$ stuffy
- for $U_1 \rightarrow F$, $U_2 \rightarrow F$, ie. $U_1 \lor U_2 \rightarrow F$, $\sigma(F) = \{\{U_1, U_2\}\}$
- disjunction of conjunctions of …
- nf: normal form, ie. nesting depth is 2
- $\text{subs}\{S_1, S_2\} = S_i$ if $S_i \subseteq S_j$ else identical
Source Definition

- For $W$, $I$, and atom $A$ (without restricting generality):
  - $\sigma(A) = \{\{A\}\}$ if $A \in I$
  - $\sigma(A) = \text{subs} \left[ \text{nf} \left\{ \{\sigma(B_{1j1}), \ldots, \sigma(B_{njn})\} \mid 1 \leq j_1 \leq m_1, \ldots, 1 \leq j_n \leq m_n \right\} \right]$ if $B_{11} \land \ldots \land B_{1m_1} \lor \ldots \lor B_{n1} \land \ldots \land B_{nm_m} \rightarrow A$
  - $\sigma(A) = \bot$ else
Illustration of Definition

- Recursive replacement of atoms by defining bodies, form dual formula, transform to DNF, remove subsumed clauses, interpret as set of sets of atoms

- Semantically, $\sigma(A)$ is set of minimal sets $M$: $I\setminus M$ does no more entail $A$ under $W$

- $U_1 \land U_2 \rightarrow F$: $\sigma(F) = \text{subs}[\text{nf}\{\{\sigma(U_1)\},\{\sigma(U_2)\}\}] = \text{subs}[\text{nf}\{\{\{U_1\}\},\{\{U_2\}\}\}] = \{\{U_1\},\{U_2\}\}$

- $U_1 \lor U_2 \rightarrow F$: $\sigma(F) = \text{subs}[\text{nf}\{\{\sigma(U_1)\},\sigma(U_2)\}\}] = \text{subs}[\text{nf}\{\{\{U_1\}\},\{U_2\}\}]) = \{\{U_1, U_2\}\}$
Inference Relation

For \( s = \{S_1, \ldots, S_m\} \) and \( t = (\{P,D\}, \{P, A\}) \):

\[
s' = \bigcup_{i=1}^{m} \{ (S_i \setminus D') \cup A \mid D' \in \sigma(D) \}
\]

\( S = S_1 \lor \ldots \lor S_m \) state set formula, whereby \( S_i \) conjunction of atoms in it.

Inductive definition of \( \vdash_T^n \) for sequences

\( \vdash() = \vdash_T^0 = \vdash \)

step n-1 to n: \( W, I \vdash (t_1, \ldots, t_m) F \) iff \( W, S' \vdash F \).
Deductions Modulo Transitions

- $W_1, U \vdash_{(t_1,t_2)} O$ in example $E_1$
- $E_2$ like $E_1$ but additional $t_3 = U \Rightarrow H$:
  $W_1, U \vdash_{(t_3,t_2)} O$, i.e. inference absorbable by additional transitions (STRIPS)

- Similar for *sets* of transitions: resulting plans (transitions) are partially ordered

- General case: $\vdash_{(T,\leq)}$ for partial order $\leq$ on set $T$
Lifting to FOL

- \( C(x) \land O(x,y) \land C(z) \Rightarrow C(x) \land O(x,z) \land C(y) \)
- \( I = \{ U(b), \exists y O(b,y), C(c) \} \)
- \( W = \{ U(x) \rightarrow C(x), \ldots \} \)
- Variables in transitions treated as parameters, recursion and negation by stratification
- Generalization to more general formulas is harder, left open for future research
Historical Background

- Leibniz conceived „possible worlds“
- Wittgenstein’s „state of affairs“
- Carnap’s „state descriptions“ (1946)
- Kripke’s semantics for modal logic (1959ff)

but modal logic studies global truths
- quasi top-down (instead of bottom-up)

and originated with different intuition
Recent History

- STRIPS 1971
  - classical deduction was never clarified and is not contained in modern systems
  - Thiébaux et al. 2003 (FF planner) introduced ⊢ within states with exponential gains but not beyond transitions as here
  - TL shares Lifschitz’s semantics with STRIPS
Recent History ctd.

- Situation calculus 1963
  - extra situation parameter
  - suffers from inferential frame problem
- Fluent calculus with language FLUX
  - like TL and its predecessors solves inferential frame problem
  - transitions on term level; not considered
- Modal approaches, active logics etc.
A Partial History Tree

- Situationskalkül 1963
  - Methoden der Vervollständigung 1987–90
  - „Successor State“-Axiome 1991
    - Fluentkalkül 1998
  - Rahmenproblem 1969
- Lineare Konnektionen 1986
  - Lineare Logik 1990
- Gleichungslogikprogramme 1990
Fluent Calculus (FC)

- state represented as term
- axioms and specification of state and actions imply constraints on solution/action and new state
- deductive plan generation
- language FLUX (Fluent Executor)
- realised in Prolog with constraints
- includes many features from KR
A Simple Office Robot

Aufgabe des Roboters sei es, mit seinen 3 Postfächern die Hauspost einzusammeln und auszuteilen.

Elementaraktionen des Roboters sind: \( fülle(x, y) \) \( x \in \{1, 2, 3\}, \ y \in \{1, \ldots, 6\} \)
\( leere(x) \) \( x \in \{1, 2, 3\} \)
\( gehe(x) \) \( x \in \{\text{vor, zurück}\} \)
Simple FLUX Program

main :- init(Z), main_loop(Z).

main_loop(Z) :-
    poss(leere(X),Z) -> execute(leere(X),Z,Z1), main_loop(Z1);
    poss(fuelle(X,Y),Z) -> execute(fuelle(X,Y),Z,Z1), main_loop(Z1);
    weiter(X,Z) -> execute(gehe(X),Z,Z1), main_loop(Z1);
    true.

weiter(X,Z) :-
    ( holds(leer(F),Z), holds(sendung(R1,R2),Z) ; holds(traegt(F,R1),Z) ),
    holds(position(R),Z), ( R < R1 -> X = vor ; X = zurueck ).

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Transition Logic
FLUX Runtime Behavior (n,3)

Sek / 100 Aktionen

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Transition Logic

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Golog Runtime Behavior \((n,3)\)
Conclusions

- Deductive approach extremely attractive
- Only competitive with a solution for both the representational and inferential frame problem (FC) and with an integration of classical $\vdash$ for compression (TL, partially FF)
- Efficient integration of TL into a theorem prover yet to be done
Midterm Perspective

- Hundreds of millions of facts in brain
- Goal: KBs with 100 mio. facts
- CYC so far 1.5 mio. facts
- Open Mind Common Sense database:
  - everyone can enter knowledge facts
  - input in natural language possible
  - within 2 years 0.5 mio. facts
- Deductive planning system on top
Trend in Full Swing . . .

- CYC (100k concepts, 10k predicates)
- Wordnet 1.6: Englisch, word/meaning
- Enterprise Ontology (modelling)
- Gene Ontology (biological concepts)
- Process Ontology (engineering)
- Cancer Ontology (100k concepts/terms)
- IEEE Standard Upper Ontology
A Practical Example

- Problem specification
  - given a standard laptop under worm attack
  - given an address for a patch program
  - goal: protected machine
  - find plan to achieve goal
- Requires several (static) reasoning steps in order to determine transitions