CMSC 251 Sample Midterm

Read all questions carefully before answering any of them. Use the space provided to answer each question. The back of the paper may be used as well if needed. If you use the back of the paper, then indicate that fact on the front. Answer all questions in blue or black pen or black pencil. You may use the information sheet provided. *Clarity and neatness count.* No books, notes, magnifying lenses, calculators, slide rules, beepers, or cellular phones are allowed in the exam room. Writing your name, section number, and student ID, *clearly*, in the space provided on each page is worth five points. Do not continue to work after time has been called.
Good luck.

Name:________________________

SID:________________________

Do not write below this line.

Name	
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Problem 1: (10 points) Show that $4n^2 + 2n - 3 = O(2n^2 - 4n + 1)$ using the definition of O given in class (do not use limits).

Problem 2: (10 points) Use the "Master Theorem" derived in class to solve exactly the following recurrence: $T(n) = 2T(n/3) + n + 4$, $T(1) = 3$. Simplify. Show your work.

Problem 3: (10 points) Consider the following recurrence for n a power of 2

$$T(n) = T(n/2) + n - 1$$

where $T(1) = 2$. Use mathematical induction to show that $T(n) = 2n - \lg n$.

Problem 4: (15 points) Consider the following recurrence, where $T(12) = 5$, and

$$T(n) \;=\; 3T(n-4) + 1 \quad \text{for } n > 12 \text{ and } n \text{ a multiple of } 4.$$

Solve the recurrence using the iteration method. Do NOT show that your solution is correct.

Problem 5: (15 points) Consider an array of size eight with the numbers $2, 4, 3, 6, 1, 5$. Assume you execute quicksort using the version of partition from CLR.

(a) What is the array after the first partition. How many comparisons did you use? How many exchanges?

(b) Show the left side after the next partition. How many comparisons did you use? How many exchanges?

(c) Show the right side after the next partition on that side. How many comparisons did you use? How many exchanges?

(d) What is the total number of comparisons in the entire algorithm? What is the total number of exchanges in the entire algorithm?

Problem 6: (10 points) Let $F(n) = \sum_{k=1}^{n}(k^2 + 2k)$. Derive a function $g(n)$ such that $F(n) = \Theta(g(n))$. Do not express $g(n)$ with any summations.

Problem 7: (15 points)

Assume that you have a list of n numbers, both positive and negative.

(a) Give an efficient algorithm to find value of the maximum contiguous sum.

(b) Analyze your algorithm.