Fall 2004 **CMSC 351: Homework 1** Clyde Kruskal

Due at the start of class Thursday, September 16, 2004.

Problem 1. Show the following. In each of (a) and (b), state specific values of the constants (e.g. c_1, c_2, N_0) you used to satisfy the conditions, and show how you arrived at the values.

(a) $2n^3 - 9n^2 + 8 = \Theta(n^3)$

(b) $7n^2 - 5n = \Theta(n^2)$.

(c) $n! \sim \sqrt{2\pi n}(n/e)^n$. Use Stirling's formula: $n! = \sqrt{2\pi n}(n/e)^n(1 + \Theta(1/n))$

(d) $n = o(n^{1.5})$

(d) $n \ln n = o(n^{1.5})$

Problem 2. Indicate, for each pair of expressions (A, B) in the table below, whether A is O,o,Ω,ω, or Θ of B. Assume that $k \geq 1$, $\epsilon > 0$, and $c > 1$ are constants. Your answer should be in the form of the table with "yes" or "no" written in each box.

	A	B	O	o	Ω	ω	Θ
a.	$\lg^k n$	n^ϵ					
b.	n^k	c^n					
c.	$\sqrt{n}$	$n^{\sin n}$					
d.	2^n	$2^{n/2}$					
e.	$n^{\lg m}$	$m^{\lg n}$					
f.	$\lg(n!)$	$\lg(n^n)$					

Problem 3. Show that for any real constants a and b, where $b > 0$,

$$(n^2 + n + a)^b = \Theta(n^{2b})$$

Problem 4. Prove whether each of (a) and (b) is true. For each, if false, give a counterexample. You may assume that for all n, $f(n) \geq 0$ and $g(n) \geq 0$.

(a) if $f(n) = O(g(n))$ then $g(n) = O(f(n))$

(b) $f(n) + g(n) = O(\max(f(n), g(n)))$

(c) $\max(f(n), g(n))) = O(f(n) + g(n))$