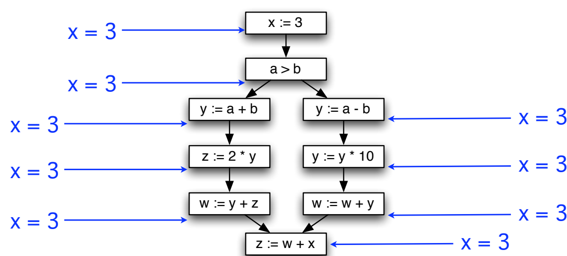
CMSC 631 – Program Analysis and Understanding
Fall 2004

Static Single Assignment Form and Dominators

Motivation

- Data flow analysis needs to represent facts at every program point
- What if
  - There are a lot of facts and
  - There are a lot of program points?
  - ⇒ potentially takes a lot of space/time
- Most likely, we're keeping track of irrelevant facts

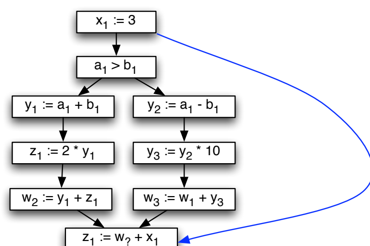
Example



Sparse Representation

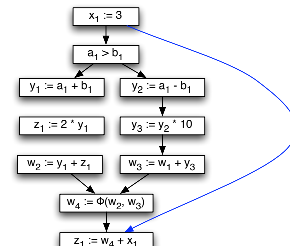
- Instead, we'd like to use a sparse representation
  - Only propagate facts about x where they're needed
- Enter *static single assignment* form
  - Each variable is defined (assigned to) exactly once
  - But may be used multiple times

Example: SSA



- Add *SSA edges* from definitions to uses
  - No intervening statements use/define variable
  - Safe to propagate only along SSA edges

What About Joins?



- Add Φ functions/nodes to model joins
  - Intuitively, takes meet of arguments
  - At code generation time, need to eliminate Φ nodes

Constant Propagation Revisited

- Initialize facts at each program point
  - C(n) := top
- Add all SSA edges to the worklist
- While the worklist isn't empty,
  - Remove an edge (x, y) from the worklist
  - C(y) := C(y) meet C(x)
  - Add SSA edges from y if C(y) changed

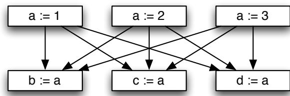
Def-Use Chains vs. SSA

- Alternative: Don't do renaming; instead, compute simple def-use chains (reaching definitions)
  - Propagate facts along def-use chains
- Drawback: Potentially quadratic size

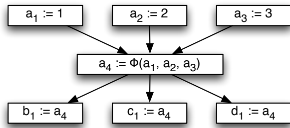
Def-Use Chains vs. SSA (cont'd)

```
case (...) of
  0: a := 1;
  1: a := 2;
  2: a := 3;
end
case (...) of
  0: b := a;
  1: c := a;
  2: d := a;
end
```

Def-Use Chains



SSA Form



Quadratic vs. (in practice) linear behavior

Conditional Constant Propagation

- So far, we assume that all branches can be taken
  - But what if some branches are never taken in practice?
    - Debugging code that can be enabled/disabled at run time
    - Macro expanded code with constants
    - Optimizations
- Idea: use constant propagation to decide which branches might be taken
  - Fits in neatly with SSA form

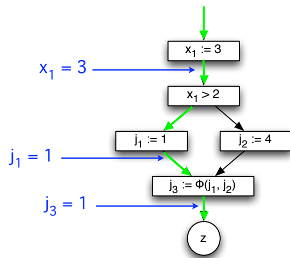
Nodes versus Edges

- So far, we've been hazy about whether data flow facts are associated with nodes or edges
  - Advantage of nodes: may be fewer of them
  - Advantage of edges: can trace differences on multiple paths to same node
- For this problem, we'll associate facts with edges

Conditional Execution

- Keep track of whether edges may be executed
  - Some may not be because they're on not-taken branch
  - Initially, assume no edges taken
  - At joins, don't propagate information from not-taken in-edges
-
- Side comment: Notice that we always, always start with the optimistic assumption
  - We need proof that a pessimistic fact holds
  - We're computing a *greatest fixpoint*

Example



Computing SSA Form

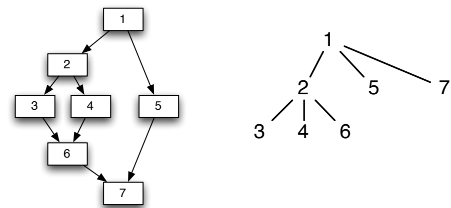
- Step 1: Compute the dominance frontier
- Step 2: Use dominance frontier to place Φ nodes
  - Naive, impractical step 2: put a Φ function for every variable at the beginning of every block
  - Better: If node X contains assignment to a, put Φ function for a in dominance frontier of X
    - Adding Φ fn may require introducing additional Φ fn
- Step 3: Rename variables so only one definition per name

Dominators

- Let X and Y be nodes in the CFG
  - Assume single entry point Entry
  -
- X *dominates* Y (written X≥Y) if
  - X appears on every path from Entry to Y
-
- Write X>Y when X dominates Y but X≠Y
  - Note ≥ is reflexive

Dominator Tree

- The dominator relationship forms a tree
  - Edge from parent to child = parent dominates child
  - Note: edges are not same as CFG edges!



Computing Dominator Tree

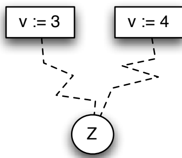
- Standard algorithm due to Lengauer and Tarjan
- Runs in time $O(E\alpha(E, N))$
  - E = # of edges, N = # of nodes
  - where $\alpha(\cdot)$ is the inverse Ackerman's function
  - Very slow growing; effectively constant in practice
- Algorithm quite difficult to understand
  - But lots of pseudo-code available

Why Are Dominators Useful?

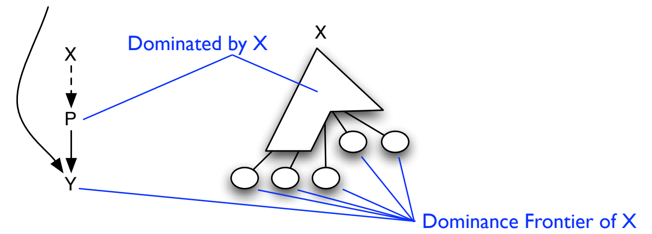
- Computing static single assignment form
- Computing control dependencies
- Identify loops in CFG
  - All nodes X dominated by entry node H, where X can reach H, and there is exactly one back edge (head dominates tail) in loop

Where do Φ Functions Go?

- We need a Φ function at node Z if
  - Two non-null CFG paths that both define v
  - Such that both paths start at two distinct nodes and end at Z



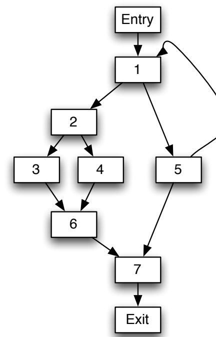
Dominance Frontiers: Illustration



Dominance Frontiers

- Y is in the dominance frontier of X iff
  - There exists a path from X to Exit through Y such that Y is the first node not strictly dominated by X
- Equivalently:
  - Y is the first node where a path from X to Exit and a path from Entry to Exit (not going through X) meet
- Equivalently:
  - X dominates a predecessor of Y
  - X does not strictly dominate Y

Example



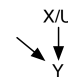
DF(1) = {1}
DF(2) = {7}
DF(3) = {6}
DF(4) = {6}
DF(5) = {1, 7}
DF(6) = {7}
DF(7) = ∅

Computing Dominance Frontiers

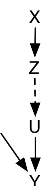
- Two components to DF(X):
  - $DF_{local}(X) = \{Y \in succ(X) \mid X \not> Y\}$
    - Any child of X not (strictly) dominated by X is in DF(X)
  -
  - Let Z be such that idom(Z) = X
    - idom(Z) is the parent of Z in the dominator tree
  - $DF_{up}(Z) = \{Y \in DF(Z) \mid X \not> Y\}$
    - Nodes from DF(Z) that are not strictly dominated by X are also in DF(X)

Why Is This Sufficient?

- Suppose $Y \in DF(X)$
  - Then there is a $U \in pred(Y)$ such that $X \geq U$, $X \not> Y$
  - If U=X, then $U \in DF_{local}(X) = \{Y \in succ(X) \mid X \not> Y\}$



  - Otherwise $U \neq X$
    - Then there is a node Z such that idom(Z)=X and $Z \geq U$
      - Possibly Z=U
    - Since $X \not> Y$, $Z \not> Y$, hence $Y \in DF(Z)$
  - Therefore $Y \in DF_{up}(Z) = \{Y \in DF(Z) \mid X \not> Y\}$



Algorithm

- Let sdom(X) = {Y | X>Y}
- In a postorder traversal on dominator tree
  - DF(X) = succ(X) - sdom(X)
    - I.e., DF(X) = DF_{local}(X)
  - For each Z such that idom(Z) = X do
    - DF(X) = DF(X) ∪ (DF(Z) - sdom(X))
    - I.e., DF(X) = DF(X) ∪ DF_{up}(Z)

Equivalent Algorithm

- In a postorder traversal on dominator tree
  - DF(X) = succ(X)
  - For each Z such that idom(Z) = X do
    - DF(X) = DF(X) ∪ DF(Z)
  - DF(X) = DF(X) - sdom(X)
  -
- See paper for another equivalent algorithm that runs in O(E+|DF|)

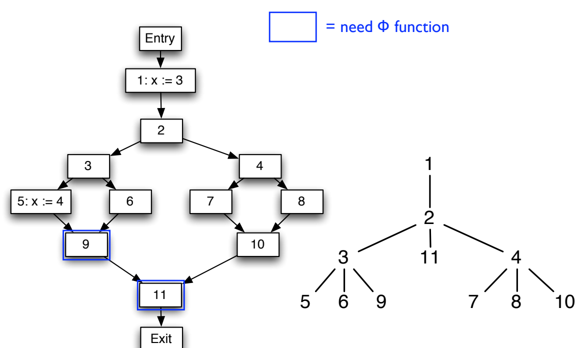
Computing SSA Form

- Step 1: Compute the dominance frontier
- Step 2: Use dominance frontier to place Φ nodes
- Step 3: Rename variables so only one definition per name

Step 2: Placing Φ Functions for v

- Let S be the set of nodes that define v
- Need to place Φ function in every node in DF(S)
  - Recall, those are all the places where the definition of v in S and some other definition of v may meet
- But a Φ function adds another definition of v!
  - v := Φ(v, ..., v)
- So, iterate
  - DF_1 = DF(S)
  - DF_{i+1} = DF(S ∪ DF_i)

Example

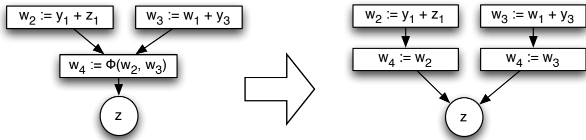


Step 3: Renaming Variables

- Top-down (DFS) traversal of dominator tree
  - At definition of v, push new # for v onto the stack
  - When leaving node with definition of v, pop stack
  - Intuitively: Works because there's a Φ function, hence a new definition of v, just beyond region dominated by definition
- Can be done in O(E+|DF|) time
  - Linear in size of CFG with Φ functions

Eliminating Φ Functions

- Basic idea: Φ represents facts that value of join may come from different paths
  - So just set along each possible path



Eliminating Φ Functions in Practice

- Copies performed at Φ fns may not be useful
  - Joined value may not be used later in the program
    - (So why leave it in?)
-
- Use dead code elimination to kill useless Φs
-
- Subsequent register allocation will map the (now very large) number of variables onto the actual set of machine register

Efficiency in Practice

- Claimed:
  - SSA grows linearly with size of program
  - No correlation between ratio and program size
- Convincing?

Arrays

- Need to handle array accesses
  - A[i] := A[j] + B[k]
- Problem: How do we know whether A[i], A[j], and B[k] are all distinct?
  - Could have A=B, e.g., foo(int A[], int B[]){} ... foo(a,a)
  - Could have i=j
- History: significant research on determining array dependencies, for parallelizing compilers

Arrays (cont'd)

- This paper's suggestion: make arrays *immutable*
  - Then don't need to worry about updates to them

```
* := A(i);          * := A(i);
A(j) := V;          A := Update(A, j, V);
* := A(k) + 2;      T := A(k);
                    * := T + 2;
```

- Update(A, j, V) makes a copy of A
  - Then try to collapse unnecessary copies
- Convincing?

Structures

- Can treat structures as sets of variables

```
* := A.f;           * := X;     // X = A.f
A.g := V;           Y := V;     // Y = A.g
* := A.f + A.g      * := X + Y
```

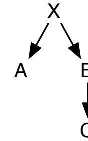
- Problems?

Pointers

- For each statement S, let
  - MustMod(S) = variables always modified by S
  - MayMod(S) = variables sometimes modified by S
    - So if v∉MayMod(S), then S must not modify v
  - MayUse(S) = variables sometimes used by S
- Then assume that statement S
  - writes to MayMod(S)
  - reads MayUse(S) ∪ (MayMod(S) - MustMod(S))
- Convincing? We'll talk more about pointers later in the course

Control Dependence

- Y is *control dependent* on X if whether Y is executed depends on a test at X



- A, B, and C are control dependent on X

Postdominators and Control Dependence

- Y *postdominates* X if every path from X to Exit contains Y
  - I.e., if X is executed, then Y is always executed
- Then, Y is control dependent on X if
  - There is a path $X \rightarrow Z_1 \rightarrow \cdots \rightarrow Z_n \rightarrow Y$ such that Y postdominates all Z_i and
  - Y does not postdominate X
  - I.e., there is some path from X on which Y is always executed, and there is some path on which Y is not executed

Dominance Frontiers, Take 2

- Postdominators are just dominators on the CFG with the edges reversed
- To see what Y is control dependent on, we want to find the Xs such that in the reverse CFG
  - There is a path $X \leftarrow Z_1 \leftarrow \cdots \leftarrow Z_n \leftarrow Y$ where
    - for all i, $Y \geq Z_i$ and
    - $Y \not> X$
- I.e., we want to find DF(Y) in the reverse CFG!