we ensure that
and setting
from the set

We are given a black-box that outputs numbers independently and uniformly at random

Suppose we only need a “weak estimate” (P2) Show that
Suppose we can draw independent random samples
we want to draw some t such samples, and output the estimate
Our main goal is that with probability at least 1 − δ, the absolute value of the difference between our estimate and

We now explore how to solve this problem. Suppose the value
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1. We have a random variable X whose distribution is some D; our aim is to estimate E[X] using “not too many” samples from D. Suppose we can draw independent random samples
we proceed
as follows, for

Let 

Suppose X is a Poisson random variable with mean λ; i.e., X takes values in the non-negative integers, with

3 (*). We are given a black-box that outputs numbers independently and uniformly at random from the set \{1, 2, \ldots, 2n\}. We use this to construct a random permutation \(\pi\) of \{1, 2, \ldots, n\} as follows. We will construct a one-to-one function \(f : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, 2n\}\), and then output the numbers in \{1, 2, \ldots, n\} in the order of their \(f(\cdot)\) values. To construct \(f\), we proceed as follows, for \(j = 1, 2, \ldots, n\): choose \(f(j)\) by repeatedly obtaining numbers from the black-box, and setting \(f(j)\) to be the first number found such that \(f(j) \neq f(i)\) for all \(i < j\). (Note that we ensure that \(f\) is one-to-one.)

Let \(X\) denote the number of calls to the black-box.
• Prove that we output a permutation that is chosen uniformly at random from the set of all $n!$ permutations.

• Find the value of $\mathbf{E}[X]$.

• Show that there is a constant $C > 0$ such that for all $n$ large enough, $\Pr[X \geq 2 \cdot \mathbf{E}[X]] \leq e^{-Cn}$. 