CMSC 858S: Algorithms in Networking

Fall 2004

Homework Assignment #3 **(will be graded)**

Due date: Beginning of class on December 9, 2004

Note: As with all homework assignments for this class, this needs to be done along with your group-mates. Please submit one final, polished set of answers for your group, and do not include unnecessary material. (For instance, don't include your preliminary work/calculations/intuition that led to the final answer.) Partial credit will be given where appropriate: if you think you have some ideas that deserve partial credit, please *itemize them concisely*, so that the grading process can be accurate.

1. Give the complete algorithm and proof for the disk-graph packet-scheduling problem (from paper [KM+04] in the class references page, discussed in class on 11/18/2004) that includes the extra $O(1 + \log(r_{max}/r_{min}))$ term. (Recall that r_{max} and r_{min} are the maximum and minimum disk-radii, respectively.) Assume that the same packet is allowed to be successfully transmitted multiple times along any edge.

2. Consider the following problem of broadcasting using minimal energy, in wireless networks. An instance of this problem consists of the following.

- A directed graph $G = (V, E)$ and a set P consisting of all power levels at which a node can transmit.
- Edge costs $c : E \rightarrow \Re^+$.
- A source node $r \in V$, and a real number B.

Node $u \in V$ transmitting at a power level $p \in P$ can reach all nodes $v \in V$ such that $(u, v) \in E$ and $c((u, v)) \leq p$. Given a power assignment vector, which defines a power level $p_v \in P$ for each node $v \in V$, define the induced graph $G' = (V, E')$, where $E' = \{(u, v) \in E : c((u, v)) \leq p_u\}$.

The question is: is there a power-assignment vector it induces a graph G' in which node r has a directed path to every other node and $\sum_{u \in V} p_u \leq B$? Show that this is NP-complete.

Hint: Try a reduction from a "covering-type" problem.

3. In this problem, our goal is to design a constant-factor approximation algorithm for a special case of the broadcast problem considered in the previous problem. Here, all nodes in V are embedded in the two-dimensional Euclidean plane, and the graph G is an undirected, complete graph. The cost $c((u, v))$ of an edge (u, v) is defined as follows:

$$c((u, v)) = d(u, v)^\alpha$$

Here, $d(u, v)$ is the Euclidean distance between u and v and $\alpha \in [2, 4]$ is a given constant. Given this network and cost model, construct a broadcast tree rooted at a source node r such that the cost of the broadcast tree is at most a constant factor times the optimal broadcast tree. In the terminology of the previous problem, the set P is the set of all non-negative reals; as in the previous problem, the graph G' can be directed. (That is, node u may choose enough power to reach v, but not necessarily vice versa.) Your algorithm should output the power levels of various nodes in the network such that every node is reachable from r in G', and the total power is at most a constant times that of the optimal power assignment. You can use the following Lemma for designing and proving the approximation guarantee of your algorithm:

Lemma: Given a set of points V within a disk of unit radius, let $G = (V, E)$ define the complete graph defined on V. Let the weight $w((u, v))$ of edge $(u, v) \in E$ be defined as follows: $w((u, v)) = d(u, v)^2$. Let C be the weight of the Minimum Spanning Tree T for G. Then, $C \leq 12$.