Algorithm Strategies

Nelson Padua-Perez
Chau-Wen Tseng

Department of Computer Science
University of Maryland, College Park

General Concepts

- **Algorithm strategy**
  - Approach to solving a problem
  - May combine several approaches

- **Algorithm structure**
  - Iterative ⇒ execute action in loop
  - Recursive ⇒ reapply action to subproblem(s)

- **Problem type**
  - Satisfying ⇒ find any satisfactory solution
  - Optimization ⇒ find best solutions (vs. cost metric)
Some Algorithm Strategies

- Recursive algorithms
- Backtracking algorithms
- Divide and conquer algorithms
- Dynamic programming algorithms
- Greedy algorithms
- Brute force algorithms
- Branch and bound algorithms
- Heuristic algorithms

Recursive Algorithm

- Based on reapplying algorithm to subproblem
- Approach
  1. Solves base case(s) directly
  2. Recurs with a simpler subproblem
  3. May need to convert solution(s) to subproblems
Backtracking Algorithm

Based on depth-first recursive search

Approach

1. Tests whether solution has been found
2. If found solution, return it
3. Else for each choice that can be made
   a) Make that choice
   b) Recur
   c) If recursion returns a solution, return it
4. If no choices remain, return failure

Some times called “search tree”

Backtracking Algorithm – Example

Find path through maze

- Start at beginning of maze
- If at exit, return true
- Else for each step from current location
  - Recursively find path
  - Return with first successful step
  - Return false if all steps fail
Backtracking Algorithm – Example

- Color a map with no more than four colors
  - If all countries have been colored return success
  - Else for each color c of four colors and country n
    - If country n is not adjacent to a country that has been colored c
      - Color country n with color c
      - Recursively color country n+1
      - If successful, return success
    - Return failure

Divide and Conquer

- Based on dividing problem into subproblems
- Approach
  1. Divide problem into smaller subproblems
     - Subproblems must be of same type
     - Subproblems do not need to overlap
  2. Solve each subproblem recursively
  3. Combine solutions to solve original problem
- Usually contains two or more recursive calls
Divide and Conquer – Examples

- **Quicksort**
  - Partition array into two parts around pivot
  - Recursively quicksort each part of array
  - Concatenate solutions

- **Mergesort**
  - Partition array into two parts
  - Recursively mergesort each half
  - Merge two sorted arrays into single sorted array

Dynamic Programming Algorithm

- Based on remembering past results

**Approach**

1. Divide problem into smaller subproblems
   - Subproblems must be of same type
   - Subproblems must overlap
2. Solve each subproblem recursively
   - May simply look up solution
3. Combine solutions into to solve original problem
4. Store solution to problem

- Generally applied to optimization problems
Fibonacci Algorithm

- Fibonacci numbers
  - fibonacci(0) = 1
  - fibonacci(1) = 1
  - fibonacci(n) = fibonacci(n-1) + fibonacci(n-2)

- Recursive algorithm to calculate fibonacci(n)
  - If n is 0 or 1, return 1
  - Else compute fibonacci(n-1) and fibonacci(n-2)
  - Return their sum

- Simple algorithm ⇒ exponential time O(2^n)

Dynamic Programming – Example

- Dynamic programming version of fibonacci(n)
  - If n is 0 or 1, return 1
  - Else solve fibonacci(n-1) and fibonacci(n-2)
    - Look up value if previously computed
    - Else recursively compute
  - Find their sum and store
  - Return result

- Dynamic programming algorithm ⇒ O(n) time
  - Since solving fibonacci(n-2) is just looking up value
Dynamic Programming – Example

Djikstra’s Shortest Path Algorithm

\[ S = \emptyset \]
\[ C[X] = 0 \]
\[ C[Y] = \infty \text{ for all other nodes} \]

while ( not all nodes in \( S \) )
  find node \( K \) not in \( S \) with smallest \( C[K] \)
  add \( K \) to \( S \)
  for each node \( M \) not in \( S \) adjacent to \( K \)
    \[ C[M] = \min \left( C[M], C[K] + \text{cost of } (K,M) \right) \]

Stores results of smaller subproblems

Greedy Algorithm

- Based on trying best current (local) choice
- Approach
  - At each step of algorithm
  - Choose best local solution
- Avoid backtracking, exponential time \( O(2^n) \)
- Hope local optimum lead to global optimum
Greedy Algorithm – Example

Kruskal’s Minimal Spanning Tree Algorithm
sort edges by weight (from least to most)
tree = ∅
for each edge (X,Y) in order
  if it does not create a cycle
     add (X,Y) to tree
   stop when tree has N–1 edges

Picks best local solution at each step

Brute Force Algorithm

- Based on trying all possible solutions

Approach
- Generate and evaluate possible solutions until
  - Satisfactory solution is found
  - Best solution is found (if can be determined)
  - All possible solutions found
    - Return best solution
    - Return failure if no satisfactory solution

- Generally most expensive approach
Brute Force Algorithm – Example

Traveling Salesman Problem (TSP)
- Given weighted undirected graph (map of cities)
- Find lowest cost path visiting all nodes (cities) once
- No known polynomial-time general solution

Brute force approach
- Find all possible paths using recursive backtracking
- Calculate cost of each path
- Return lowest cost path

Requires exponential time $O(2^n)$

Branch and Bound Algorithm

Based on limiting search using current solution

Approach
- Track best current solution found
- Eliminate partial solutions that can not improve upon best current solution
- Reduces amount of backtracking

Not guaranteed to avoid exponential time $O(2^n)$
Branch and Bound – Example

- Branch and bound algorithm for TSP
  - Find possible paths using recursive backtracking
  - Track cost of best current solution found
  - Stop searching path if cost > best current solution
  - Return lowest cost path
- If good solution found early, can reduce search
- May still require exponential time $O(2^n)$

Heuristic Algorithm

- Based on trying to guide search for solution
- Heuristic ⇒ “rule of thumb”
- Approach
  - Generate and evaluate possible solutions
    - Using “rule of thumb”
    - Stop if satisfactory solution is found
- Can reduce complexity
- Not guaranteed to yield best solution
Heuristic Algorithm – Example

- Heuristic algorithm for TSP
  - Find possible paths using recursive backtracking
    - Search 2 lowest cost edges at each node first
  - Calculate cost of each path
  - Return lowest cost path from first 100 solutions
- Not guaranteed to find best solution
- Heuristics used frequently in real applications

Summary

- Wide range of strategies
- Choice depends on
  - Properties of problem
  - Expected problem size
  - Available resources