Fall 2005 **CMSC 451: Homework 0** Samir Khuller

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Problem 1. For each pair of expressions (A, B) below, indicate whether A is O, o, Ω, ω, or Θ of B. Note that zero, one or more of these relations may hold for a given pair; list all correct ones.

	A	B
(a)	n^{100}	2^n
(b)	$(\log n)^{12}$	$\sqrt{n}$
(c)	10^n	100^n
(d)	$n^{\log n}$	$(\log n)^n$
(e)	$\log(n!)$	$n \log n$

Problem 2. Prove by induction

$$\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}.$$

Problem 3. Assume every node of a tree with n nodes has either 0 or 3 children. How many leaves does the tree have? Prove your answer.

Problem 4. Solve the recurrence

$$T(n) = \begin{cases} T(n/5) + 2 & n > 1 \\ 3 & \text{otherwise} \end{cases}$$

assuming n is a power of 5. Show your calculations.

Problem 5. A coin is tossed n times, each time with an independent probability p of coming up heads and $1 - p$ of coming up tails. Let H be the number of heads occurring. What is

(a) $E[H]$, the expected number of heads?

(b) $V[H]$, the variance of H?

(c) the standard deviation of H?

(d) the probability that $H > 2$?

Show your calculations.