

Due in class: Oct 13.

- (1) Given a directed graph  $G = (V, E)$  we wish to compute for each edge in the graph, if it belongs to a cycle. How can we do this in time  $O(|E| + |V|)$ ?
- (2) Let  $G = (V, E)$  be an undirected graph. Design a depth-first search algorithm to determine whether it is possible to direct the edges of  $G$  such that the indegree of every vertex is at least 1. If it is possible to do so, then the algorithm should show a way to do it. Justify the correctness of your algorithm and derive its running time. (Hint: A graph that is a tree never has such an orientation. A graph that has a cycle ALWAYS has such an orientation.)
- (3) Show that in any simple graph of  $n$  vertices ( $n \geq 2$ ) there are always at least two vertices that have the same degree (i.e., there exist  $u, v$  such that  $\deg(u) = \deg(v)$ ).
- (4) Consider the following problem: There is a set of movies  $M_1, M_2, \dots, M_k$ . There is also a set of customers, with each one indicating the two movies they would like to see this weekend. (Assume that customer  $i$  specifies a subset  $S_i$  of the two movies that he/she would like to see. Movies are shown on saturday evening and sunday evening. Multiple movies may be screened at the same time.)

You need to decide which movies should be televised on saturday, and which movies should be televised on sunday, so that every customer gets to see the two movies they desire. The question is: is there a schedule so that each movie is shown at most once? Design an efficient algorithm to find such a schedule if one exists?

(One naive solution would be to show all movies on both days, so that each customer can see one desired movie on each day. We would need  $k$  channels if  $k$  is the number of movies in this solution – hence if there is a solution in which each movie is shown at most once, we would like to find it.)

- (5) Consider a rooted DAG (directed, acyclic graph with a vertex – the root – that has a path to all other vertices). Give a linear time ( $O(|V| + |E|)$ ) algorithm to find the length of the longest simple path from the root to each of the other vertices. (The length of a path is the number of edges on it.)