Fall 2005 **CMSC 451: Homework 6** Samir Khuller

Due in class: Nov 17.

(1) Write out the proof in detail that justifies the correctness of the method described in class for saving space in the Floyd-Warshall algorithm. (Essentially, to define the matrix d^k, we use the same space as for d^{k-1}.)

(2) You are planning a long road trip to Los Angeles. We wish to find the shortest route to LA, but are not willing to spend more than C \$'s in tolls.

We can model this problem as follows: given a directed graph, where each edge has a *length* and a *cost* we wish to find the shortest length path from s to t (given an arbitrary pair of vertices) such that the total *cost* of the path does not exceed C. Moreover, we will assume that the cost of an edge is an integer. You have to design an algorithm with running time $O(f(n,m)C)$ where $f(n,m)$ is a polynomial function of the number of vertices n and the number of edges m in the network.

The input to the problem is: a directed graph G represented as an adjacency list a pair of vertices s, t, and an integer C. You have to output the shortest length path from s to t which has cost at most C.

(3) Problem 22 (page 330).

(4) Consider the problem of making change for n cents using the smallest number of coins. Assume that each coin's value is an integer.

- Describe a greedy algorithm to make change consisting of quarters, dimes, nickels and pennies. Prove that your algorithm yields an optimal solution.
- Suppose that the available coins are powers of c, i.e., the denominations are $c^0, \ldots, c^k$ for some integers $c > 1$ and $k \geq 1$. Show that the greedy algorithm gives an optimal solution.
- Give a set of coin denominations for which the greedy algorithm is not optimal. You should include pennies, so that there is a solution for every value n.