Fall 2005 **CMSC 451: Homework 8** Samir Khuller

Due in class: Dec 8. No late homeworks will be accepted since the solutions will be posted on Dec 8th itself.

Please fill out the Course Evaluation immediately.

When you turn in your homework, sign a statement to the effect that you have done this. Your feedback is very important to both the University, and me personally.

(1)
- Suppose we know that problem X is NP-complete. Suppose we discover a polynomial time algorithm for X. Would that imply that the SATISFIABILITY problem can be solved in polynomial time? Explain your answer.
- Suppose we know that problem X belongs to NP. Suppose we discover a polynomial time algorithm for X. Would that imply that the SATISFIABILITY problem can be solved in polynomial time? Explain your answer.
- Suppose Suppose we discover an $O(n^3)$ algorithm for SATISFIABILITY. Would that imply that every problem in NP can be solved in $O(n^3)$ time? Why or why not?

(2) The graph-3-coloring problem is defined as follows: Given a graph $G = (V, E)$ and three colors $\{1, 2, 3\}$, is there a way to assign a color to each node so that adjacent nodes always have different colors? Note that we can only use three colors.

Assume that Graph-3-coloring is NP-complete. Prove that graph-4-coloring is NP-complete.

(3) Give a polynomial reduction from Dominating Set to Set Cover. (Set cover is defined in Chapter 8 in the book.)

(4) HAMILTONIAN PATH PROBLEM: given a directed graph, does it contain a path that starts at some vertex and goes to some other vertex, going through each remaining vertex exactly once.

HAMILTONIAN CYCLE PROBLEM: given a directed simple graph, does it contain a directed simple cycle that goes through each vertex exactly once.

Assume that the HAMILTONIAN PATH PROBLEM is known to be NP-complete. Given this assumption, prove that the HAMILTONIAN CYCLE PROBLEM is NP-complete for directed graphs. (Show that the HAMILTONIAN CYCLE PROBLEM is in NP, and is also NP-hard.)