

CMSC 132: Object-Oriented Programming II



Hashing

Department of Computer Science
University of Maryland, College Park

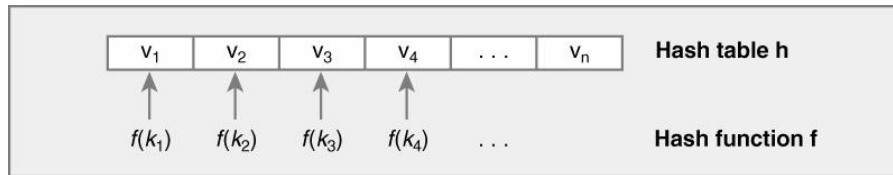
Overview

- **Hashing**
 - Scattering Hash Values
 - Hash Function
- **Hash Tables**
 - Open Addressing
 - Chaining

Hashing

■ Approach

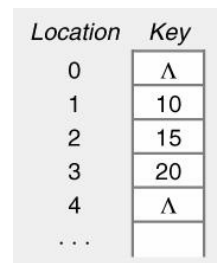
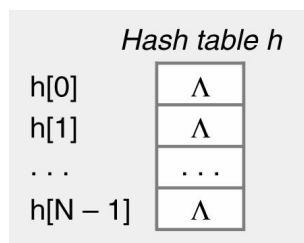
- Use **hash function** to convert key into number (**hash value**) used as index in **hash table**



Hashing

■ Hash Table

- Array indexed using hash values
- Hash table A with size N
- Indices of A range from 0 to $N-1$
- Store in $A[\text{hashValue} \% N]$



Hash Function

- Function for converting key into hash value
- For Java
 - Hash value \Rightarrow 32-bit signed int
 - Default hash function \Rightarrow int hashCode()
- For hash table of size N
 - Must reduce hash value to $0..N - 1$

Scattering Hash Values

- Should scatter hash values uniformly across range of possible values
 - Hash(<everything>) = 0
 - Satisfies definition of hash function
 - But not very useful (all keys at same location)
- Could use $\text{Math.abs}(\text{key.hashCode}() \% N)$
 - Might not distribute values well
 - Particularly if N is a power of 2

Scattering Hash Values

- **Multiplicative congruency method**
 - Produces good hash values
 - Hash value = `Math.abs((a * key.hashCode()) % N)`
 - Where
 - N is table size
 - a, N are large primes

Beware of % (Modulo Operator)

- The % operator is integer remainder

$$x \% y == x - y * (x / y)$$

- Not what mathematicians expect

<code>3/2 == 1</code>	<code>3%2 == 1</code>
<code>2/2 == 1</code>	<code>2%2 == 0</code>
<code>1/2 == 0</code>	<code>1%2 == 1</code>
<code>0/2 == 0</code>	<code>0% 2 == 0</code>
<code>(- 1)/2 == 0</code>	<code>(- 1)%2 == - 1</code>
<code>(- 2)/2 == - 1</code>	<code>(- 2)%2 == 0</code>
<code>(- 3)/2 == - 1</code>	<code>(- 3)%2 == - 1</code>

- Use `Math.abs(x % N)`
 - Rather than `Math.abs(x) % N`

Art and Magic of hashCode()

- There is no “right” hashCode function
 - Art involved in finding good hashCode function
 - Also for finding hashCode to hashBucket function
- From java.util.HashMap

```
static int hashBucket(Object x, int N) {  
    int h = x.hashCode();  
    h += ~(h << 9);  
    h ^= (h >>> 14);  
    h += (h << 4);  
    h ^= (h >>> 10);  
    return Math.abs(h % N);  
}
```

Hash Function

■ Example

```
hashCode("apple") = 5  
hashCode("watermelon") = 3  
hashCode("grapes") = 8  
hashCode("kiwi") = 0  
hashCode("strawberry") = 9  
hashCode("mango") = 6  
hashCode("banana") = 2
```

■ Perfect hash function

- Unique values for each key

0	kiwi
1	
2	banana
3	watermelon
4	
5	apple
6	mango
7	
8	grapes
9	strawberry

Hash Function

■ **Suppose now**

- hashCode("apple") = 5
- hashCode("watermelon") = 3
- hashCode("grapes") = 8
- hashCode("kiwi") = 0
- hashCode("strawberry") = 9
- hashCode("mango") = 6
- hashCode("banana") = 2
- hashCode("orange") = 3

0	kiwi
1	
2	banana
3	watermelon
4	
5	apple
6	mango
7	
8	grapes
9	strawberry

■ **Collision**

- Same hash value for multiple keys

Types of Hash Tables

■ **Open addressing**

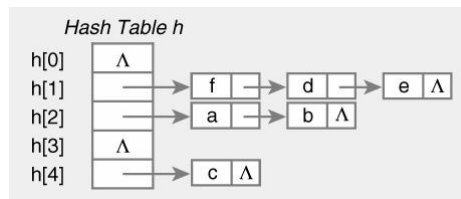
- Store objects in each table entry

Hash table h

h[0]	(k ₄ , v ₄)
h[1]	Λ
h[2]	Λ
h[3]	(k ₁ , v ₁)
h[4]	(k ₃ , v ₃)
h[5]	(k ₂ , v ₂)

■ **Chaining (bucket hashing)**

- Store lists of objects in each table entry



Open Addressing Hashing

■ Approach

- Hash table contains objects
- Probe \Rightarrow examine table entry
- Collision
 - Move **K** entries past current location
 - Wrap around table if necessary
- Find location for **X**
 1. Examine entry at $A[\text{key}(X)]$
 2. If entry = **X**, found
 3. If entry = empty, **X** not in hash table
 4. Else increment location by **K**, repeat

Open Addressing Hashing

■ Approach

- Linear probing
 - $K = 1$
 - May form **clusters** of contiguous entries
- Deletions
 - Find location for **X**
 - If **X** inside cluster, leave **non-empty** marker
- Insertion
 - Find location for **X**
 - Insert if **X** not in hash table
 - Can insert **X** at first non-empty marker

Open Addressing Example

■ **Hash codes**

- H(A) = 6 H(C) = 6
- H(B) = 7 H(D) = 7

■ **Hash table**

- Size = 8 elements
- Λ = empty entry
- * = non-empty marker

■ **Linear probing**

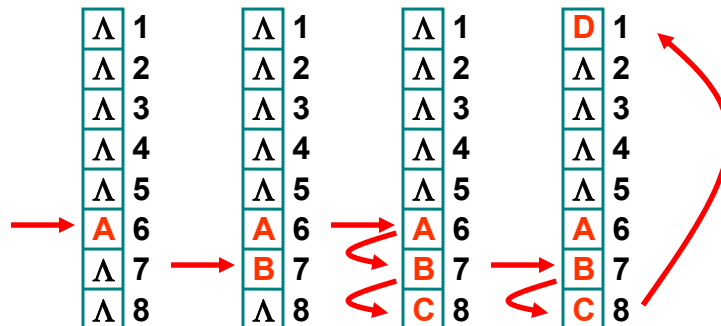
- Collision \Rightarrow move 1 entry past current location

Λ	1
Λ	2
Λ	3
Λ	4
Λ	5
Λ	6
Λ	7
Λ	8

Open Addressing Example

■ **Operations**

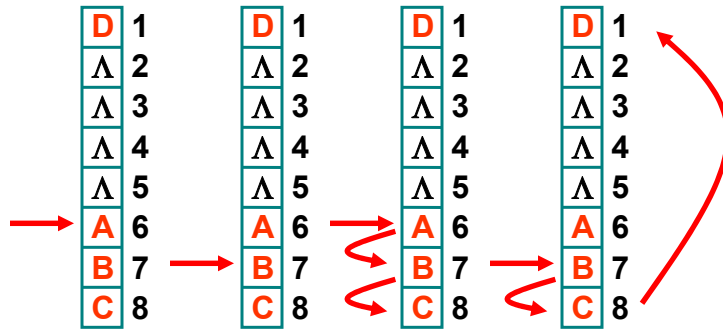
- Insert A, Insert B, Insert C, Insert D



Open Addressing Example

■ Operations

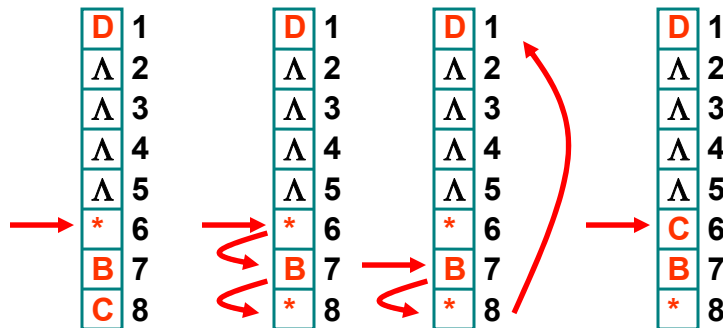
- Find A, Find B, Find C, Find D



Open Addressing Example

■ Operations

- Delete A, Delete C, Find D, Insert C



Efficiency of Open Hashing

- Load factor = entries / table size
- Hashing is efficient for load factor < 90%

α	Number of Comparisons	Approximate Behavior	(Table Size $N = 100$)
0.1	1.06	O(1)	
0.2	1.13		
0.3	1.21		
0.4	1.33		
0.5	1.50		
0.6	1.75		
0.7	2.17	O(log N)	
0.8	3.00		
0.9	5.50	O(N)	
0.95	10.5		
0.98	26.5		
0.99	50.5		

Chaining (Bucket Hashing)

- Approach
 - Hash table contains lists of objects
 - Find location for X
 - Find hash code key for X
 - Examine list at table entry $A[\text{key}]$
 - Collision
 - Multiple entries in list for entry

Chaining Example

■ Hash codes

■ $H(A) = 6$ $H(C) = 6$

■ $H(B) = 7$ $H(D) = 7$

■ Hash table

■ Size = 8 elements

■ Λ = empty entry

1	Λ
2	Λ
3	Λ
4	Λ
5	Λ
6	Λ
7	Λ
8	Λ

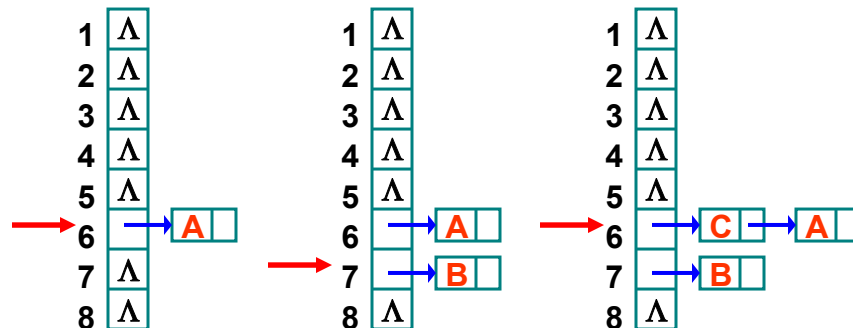
Chaining Example

■ Operations

■ Insert A,

Insert B,

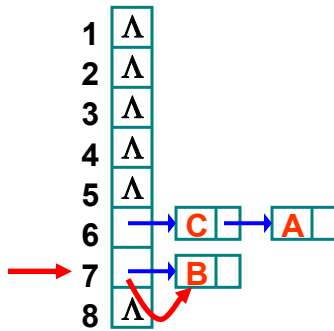
Insert C



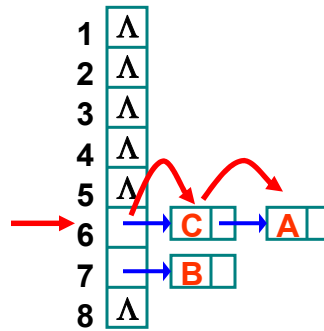
Chaining Example

■ Operations

■ Find B,



Find A



Efficiency of Chaining

■ Load factor = entries / table size

■ Average case

- Evenly scattered entries
- Operations = $O(\text{load factor})$

■ Worse case

- Entries mostly have same hash value
- Operations = $O(\text{entries})$

Hashing in Java

■ Collections

- `HashMap` & `HashSet` implement hashing

■ Objects

- Built-in support for hashing
 - `boolean equals(Object o)`
 - `int hashCode()`
- Can override with own definitions
- Must be careful to support Java contract
 - if `a.equals(b) == true`
 - then `a.hashCode() == b.hashCode()` must be true