CMSC 132:
Object-Oriented Programming II

Sorting

Department of Computer Science
University of Maryland, College Park

Overview

- Comparison sort
  - Bubble sort
  - Selection sort
  - Tree sort
  - Heap sort
  - Quick sort
  - Merge sort
- Linear sort
  - Counting sort
  - Bucket (bin) sort
  - Radix sort

Bubble sort, Selection sort: $O(n^2)$

Tree sort, Heap sort, Quick sort, Merge sort: O(n log(n))

Counting sort, Bucket (bin) sort, Radix sort: O(n)

Sorting

- Goal
  - Arrange elements in predetermined order
    - Based on key for each element
  - Derived from ability to compare two keys by size
- Properties
  - Stable ⇒ relative order of equal keys unchanged
    - Stable: 3, 1, 4, 3, 3, 2 → 1, 2, 3, 3, 3, 4
    - Unstable: 3, 1, 4, 3, 3, 2 → 1, 2, 3, 3, 3, 4
  - In-place ⇒ uses only constant additional space
  - External ⇒ can efficiently sort large # of keys

Sorting

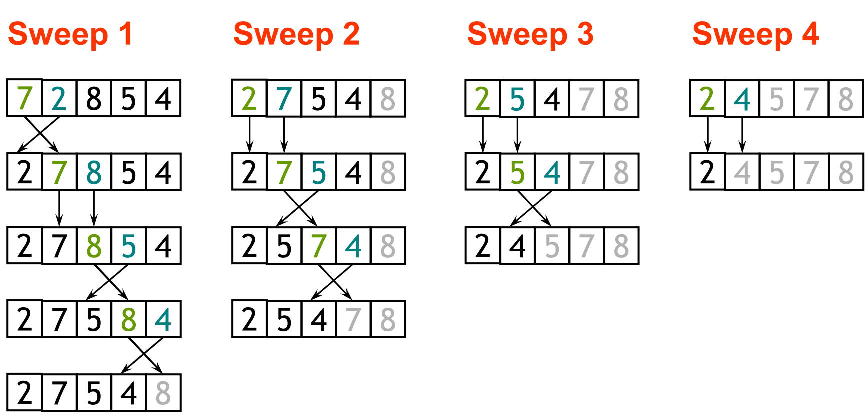
- Comparison sort
  - Only uses pairwise key comparisons
  - Proven lower bound of O(n log(n))
- Linear sort
  - Uses additional properties of keys

Bubble Sort

- **Approach**
  1. **Iteratively sweep through shrinking portions of list**
  2. **Swap element x with its right neighbor if x is larger**

- **Performance**
  - **O(n^2) average / worst case**

Bubble Sort Example



Bubble Sort Code

```
void bubbleSort(int[ ] a) {
   int outer, inner;
   for (outer = a.length - 1; outer > 0; outer--)
      for (inner = 0; inner < outer; inner++)
         if (a[inner] > a[inner + 1])
            swap(a, inner, inner+1);
}

void swap(int a[ ], int x, int y) {
   int temp = a[x];
   a[x] = a[y];
   a[y] = temp;
}
```

Sweep through array

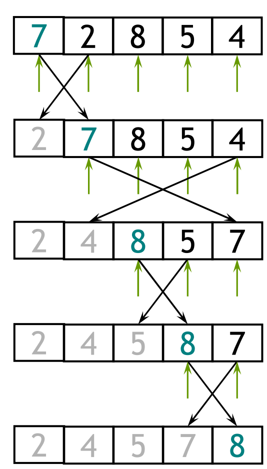
Swap with right neighbor if larger

Swap array elements at positions x & y

Selection Sort

- **Approach**
  1. Iteratively sweep through shrinking portions of list
  2. Select smallest element found in each sweep
  3. Swap smallest element with front of current list
- **Performance**
  - O(n^2) average / worst case

- **Example**



Selection Sort Code

```
void selectionSort(int[] a) {
      int outer, inner, min;
      for (outer = 0; outer < a.length - 1; outer++) {
         min = outer;
         for (inner = outer + 1; inner < a.length; inner++) {
            if (a[inner] < a[min]) {
               min = inner;
            }
         }
         swap(a, outer, min);
      }
}
```

Sweep through array

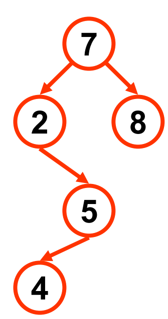
Find smallest element

Swap with smallest element found

Tree Sort

- Approach
  1. Insert elements in binary search tree
  2. List elements using inorder traversal
- Performance
  - Binary search tree
    - O(n log(n)) average case
    - O(n^2) worst case
  - Balanced binary search tree
    - O(n log(n)) average / worst case
- Example

  Binary search tree

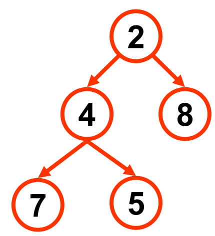


{ 7, 2, 8, 5, 4 }

Heap Sort

- **Approach**
  1. Insert elements in heap
  2. Remove smallest element in heap, repeat
  3. List elements in order of removal from heap
- **Performance**
  - O(n log(n)) average / worst case
- **Example**

  Heap



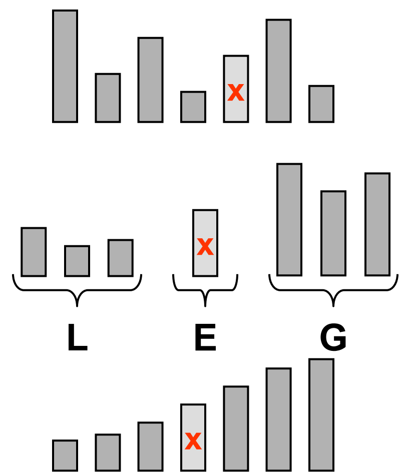
{ 7, 2, 8, 5, 4 }

Quick Sort

- **Approach**
  1. Select pivot value (near median of list)
  2. Partition elements (into 2 lists) using pivot value
  3. Recursively sort both resulting lists
  4. Concatenate resulting lists
  - For efficiency pivot needs to partition list evenly
- **Performance**
  - O(n log(n)) average case
  - O(n^2) worst case

Quick Sort Algorithm

1. If list below size K
   - Sort w/ other algorithm
2. Else pick pivot x and partition S into
   - L elements < x
   - E elements = x
   - G elements > x
3. Quicksort L & G
4. Concatenate L, E & G
   - If not sorting in place



13

Quick Sort Code

```
void quickSort(int[] a, int x, int y) {
      int pivotIndex;
      if ((y - x) > 0) {
         pivotIndex = partionList(a, x, y);
         quickSort(a, x, pivotIndex - 1);
         quickSort(a, pivotIndex+1, y);
      }
}
```

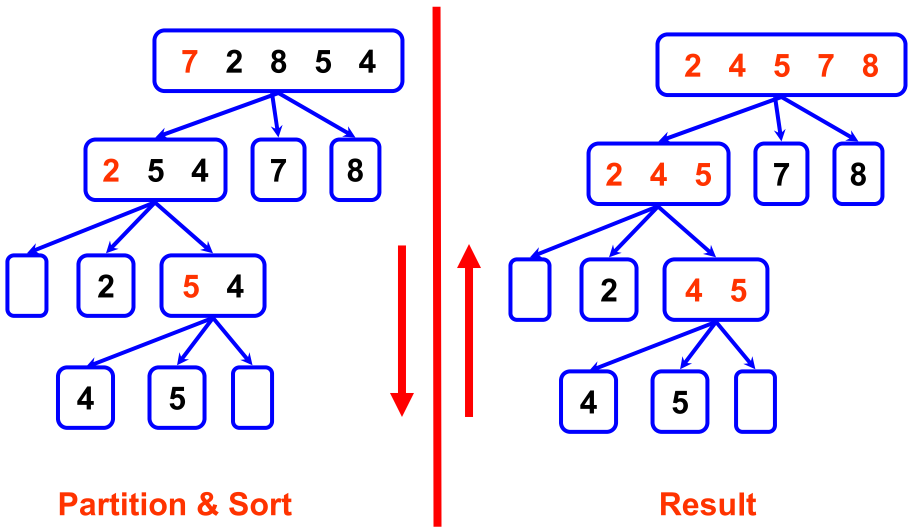
Lower end of array region to be sorted

Upper end of array region to be sorted

```
int partionList(int[] a, int x, int y) {
       … // partitions list and returns index of pivot
}
```

14

Quick Sort Example



Quick Sort Code

```
int partitionList(int[] a, int x, int y) {
      int pivot = a[x];
      int left = x;
      int right = y;
      while (left < right) {
        while ((a[left] < pivot) && (left < right))
           left++;
        while (a[right] > pivot)
           right--;
         if (left < right)
           swap(a, left, right);
       }
       swap(a, x, right);
       return right;
}
```

Use first element as pivot

Partition elements in array relative to value of pivot

Place pivot in middle of partitioned array

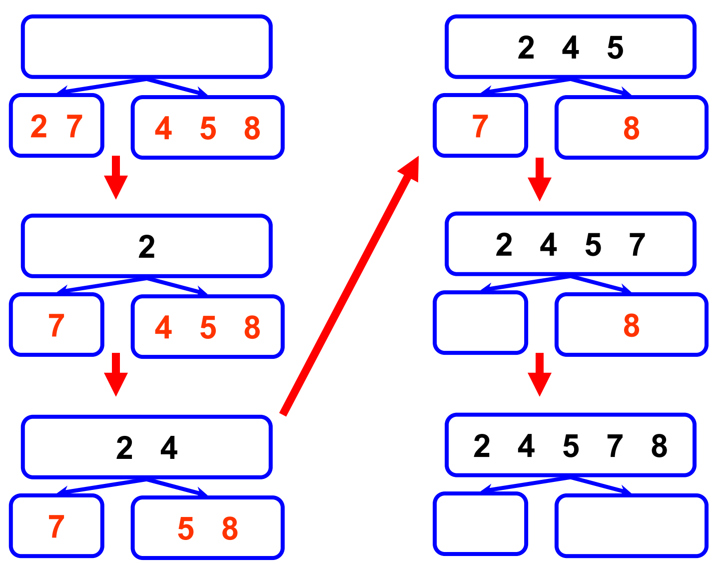
return index of pivot

Merge Sort

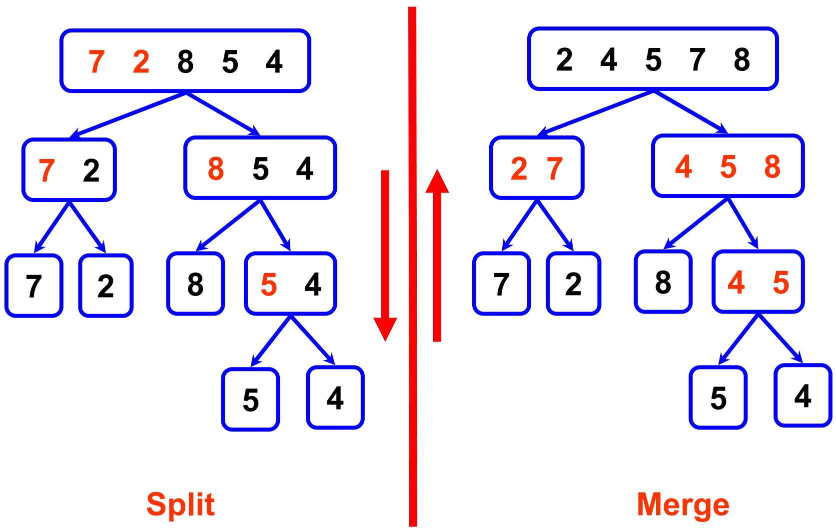
- **Approach**
  1. Partition list of elements into 2 lists
  2. Recursively sort both lists
  3. Given 2 sorted lists, merge into 1 sorted list
     a) Examine head of both lists
     b) Move smaller to end of new list

- **Performance**
  - O(n log(n)) average / worst case

Merge Example



Merge Sort Example



Merge Sort Code

```
void mergeSort(int[] a, int x, int y) {
      int mid = (x + y) / 2;
      if (y == x) return;
      mergeSort(a, x, mid);
      mergeSort(a, mid+1, y);
      merge(a, x, y, mid);
}
void merge(int[] a, int x, int y, int mid) {
      … // merges 2 adjacent sorted lists in array
}
```

Lower end of array region to be sorted

Upper end of array region to be sorted

Merge Sort Code

```
void merge (int[] a, int x, int y, int mid) {
   int size = y - x;
   int left = x;
   int right = mid+1;
   int[] tmp; int j;
   for (j = 0; j < size; j++) {
      if (left > mid) tmp[j] = a[right++];
      else if (right > y) || (a[left] < a[right])
         tmp[j] = a[left++];
      else tmp[j] = a[right++];
      }
   for (j = 0; j < size; j++)
      a[x+j] = tmp[j];
}
```

Upper end of 1st array region

Lower end of 1st array region

Upper end of 2nd array region

Copy smaller of two elements at head of 2 array regions to tmp buffer, then move on

Copy merged array back

21

Counting Sort

- **Approach**
  1. Sorts keys with values over range 0..k
  2. Count number of occurrences of each key
  3. Calculate # of keys ≤ each key
  4. Place keys in sorted location using # keys counted
     - If there are x keys ≤ key y
     - Put y in x^{th} position
     - Decrement x in case more instances of key y
- **Properties**
  - O(n + k) average / worst case

22

Counting Sort Example

- **Original list**

0	1	2	3	4
7	2	8	5	4

- **Count**

0	1	2	3	4	5	6	7	8
0	0	1	0	1	1	0	1	1

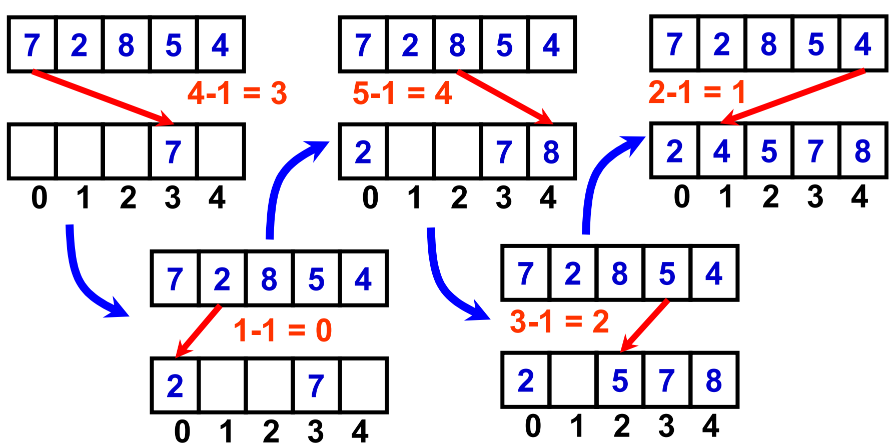
- **Calculate # keys ≤ value**

0	1	2	3	4	5	6	7	8
0	0	1	1	2	3	3	4	5

Counting Sort Example

- **Assign locations**

0	1	2	3	4	5	6	7	8
0	0	1	1	2	3	3	4	5



Counting Sort Code

```
void countSort(int[] a, int k) {  // keys have value 0…k
   int[] b; int[] c; int i;
   for (i = 0; i ≤ k; i++)           // initialize counts
      c[i] = 0;
   for (i = 0; i < a.size(); i++)  // count # keys
      c[a[i]]++;
   for (i = 1; i ≤ k; i++)          // calculate # keys ≤ value i
      c[i] = c[i] + c[i-1]
   for (i = a.size()-1; i > 0; i--) {
      b[c[a[i]]-1] = a[i];          // move key to location
      c[a[i]]--;                     // decrement # keys ≤ a[i]
   }
   for (i = 0; i < a.size(); i++)  // copy sorted list back to a
      a[i] = b[i];
}
```

25

Bucket (Bin) Sort

- **Approach**
  1. **Divide key interval into k equal-sized subintervals**
  2. **Place elements from each subinterval into bucket**
  3. **Sort buckets (using other sorting algorithm)**
  4. **Concatenate buckets in order**
- **Properties**
  - **Pick large k so can sort n / k elements in O(1) time**
  - **O(n) average case**
  - **O(n^2) worst case**
    - **If most elements placed in same bucket and sorting buckets with O(n^2) algorithm**

26

Bucket Sort Example

1. Original list
   - 623, 192, 144, 253, 152, 752, 552, 231
2. Bucket based on 1st digit, then sort bucket
   - 192, 144, 152 ⇒ 144, 152, 192
   - 253, 231 ⇒ 231, 253
   - 552 ⇒ 552
   - 623 ⇒ 623
   - 752 ⇒ 752
3. Concatenate buckets
   - 144, 152, 192 | 231, 253 | 552 | 623 | 752

27

Radix Sort

- Approach
  1. Decompose key C into components C_1, C_2, … C_d
     - Component d is least significant
     - Each component has values over range 0..k
  2. For each key component i = d down to 1
     - Apply linear sort based on component C_i (sort must be stable)
  - Example key components
    - Letters (string), digits (number)
- Properties
  - $O(d \times (n+k)) \approx O(n)$ average / worst case

28

Radix Sort Example

1. **Original list**
   - 623, 192, 144, 253, 152, 752, 552, 231
2. **Sort on 3rd digit** (counting sort from 0-9)
   - 231, 192, 152, 752, 552, 623, 253, 144
3. **Sort on 2nd digit** (counting sort from 0-9)
   - 623, 231, 144, 152, 752, 552, 253, 192
4. **Sort on 1st digit** (counting sort from 0-9)
   - 144, 152, 192, 231, 253, 552, 623, 752

Compare with: counting sort from 192-752

29

Sorting Properties

Name	Comparison Sort	Avg Case Complexity	Worst Case Complexity	In Place	Can be Stable
Bubble	√	$O(n^2)$	$O(n^2)$	√	√
Selection	√	$O(n^2)$	$O(n^2)$	√	√
Tree	√	O(n log(n))	$O(n^2)$		
Heap	√	O(n log(n))	O(n log(n))		
Quick	√	O(n log(n))	$O(n^2)$	√	
Merge	√	O(n log(n))	O(n log(n))		√
Counting		O(n)	O(n)		√
Bucket		O(n)	$O(n^2)$		√
Radix		O(n)	O(n)		√

30

Sorting Summary

- Many different sorting algorithms
- Complexity and behavior varies
- Size and characteristics of data affect algorithm