CMSC 132:
Object-Oriented Programming II

Advanced Tree Structures

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Overview

- Binary trees
  - Balance
  - Rotation

- Multi-way trees
  - Search
  - Insert

- Indexed tries
Tree Balance

- **Degenerate**
  - Worst case
  - Search in $O(n)$ time

- **Balanced**
  - Average case
  - Search in $O(\log(n))$ time
Question
- Can we keep tree (mostly) balanced?

Self-balancing binary search trees
- AVL trees
- Red-black trees

Approach
- Select invariant (that keeps tree balanced)
- Fix tree after each insertion / deletion
  - Maintain invariant using rotations
- Provides operations with $O(\log(n))$ worst case
AVL Trees

Properties
- Binary search tree
- Heights of children for node differ by at most 1

Example

Heights of children shown in red
AVL Trees

History

- Discovered in 1962 by two Russian mathematicians, Adelson-Velskii & Landis

Algorithm

1. Find / insert / delete as a binary search tree
2. After each insertion / deletion
   a) If height of children differ by more than 1
   b) Rotate children until subtrees are balanced
   c) Repeat check for parent (until root reached)
Red-black Trees

Properties

- Binary search tree
- Every node is red or black
- The root is black
- Every leaf is black
- All children of red nodes are black
- For each leaf, same # of black nodes on path to root

Characteristics

- Properties ensures no leaf is twice as far from root as another leaf
Red-black Trees

Example
Red-black Trees

- **History**
  - Discovered in 1972 by Rudolf Bayer

- **Algorithm**
  - Insert / delete may require complicated bookkeeping & rotations

- **Java collections**
  - TreeMap, TreeSet use red-black trees
Tree Rotations

Changes shape of tree
- Move nodes
- Change edges

Types
- Single rotation
  - Left
  - Right
- Double rotation
  - Left-right
  - Right-left
Tree Rotation Example

Single right rotation
Tree Rotation Example

Single right rotation

Node 4 attached to new parent
Example – Single Rotations

single left rotation

single right rotation
Example – Double Rotations

right-left double rotation

left-right double rotation
Multi-way Search Trees

Properties

- Generalization of binary search tree
- Node contains 1…k keys (in sorted order)
- Node contains 2…k+1 children
- Keys in j\textsuperscript{th} child < j\textsuperscript{th} key < keys in (j+1)\textsuperscript{th} child

Examples
Types of Multi-way Search Trees

- 2-3 tree
  - Internal nodes have 2 or 3 children

- Index search trie
  - Internal nodes have up to 26 children (for strings)

- B-tree
  - \( T = \) minimum degree
  - Non-root internal nodes have \( T-1 \) to \( 2T-1 \) children
  - All leaves have same depth
Multi-way Search Trees

Search algorithm
1. Compare key $x$ to 1…$k$ keys in node
2. If $x =$ some key then return node
3. Else if ($x < \text{key } j$) search child $j$
4. Else if ($x > \text{all keys}$) search child $k+1$

Example
Search(17)
Multi-way Search Trees

Insert algorithm
1. Search key $x$ to find node $n$
2. If ( $n$ not full ) insert $x$ in $n$
3. Else if ( $n$ is full )
   a) Split $n$ into two nodes
   b) Move middle key from $n$ to $n$’s parent
   c) Insert $x$ in $n$
   d) Recursively split $n$’s parent(s) if necessary
Multi-way Search Trees

Insert Example (for 2-3 tree)

Insert( 4 )
Multi-way Search Trees

Insert Example (for 2-3 tree)

- Insert( 1 )

Split node

Split parent
B-Trees

Characteristics

- Height of tree is $O(\log_T(n))$
- Reduces number of nodes accessed
- Wasted space for non-full nodes

Popular for large databases

- 1 node = 1 disk block
- Reduces number of disk blocks read
Indexed Search Tree (Trie)

- Special case of tree

- Applicable when
  - Key C can be decomposed into a sequence of subkeys $C_1, C_2, \ldots, C_n$
  - Redundancy exists between subkeys

- Approach
  - Store subkey at each node
  - Path through trie yields full key

- Example
  - Huffman tree
Tries

- Useful for searching strings
  - String decomposes into sequence of letters
  - Example
    - “ART” ⇒ “A” “R” “T”
- Can be very fast
  - Less overhead than hashing
- May reduce memory
  - Exploiting redundancy
- May require more memory
  - Explicitly storing substrings
Types of Tries

- **Standard**
  - Single character per node

- **Compressed**
  - Eliminating chains of nodes

- **Compact**
  - Stores indices into original string(s)

- **Suffix**
  - Stores all suffixes of string
Standard Tries

Approach
- Each node (except root) is labeled with a character
- Children of node are ordered (alphabetically)
- Paths from root to leaves yield all input strings

Trie for Morse Code
Standard Trie Example

For strings

\{ a, an, and, any, at \}
Standard Trie Example

For strings

{ bear, bell, bid, bull, buy, sell, stock, stop }
Standard Tries

Node structure
- Value between 1…m
- Reference to m children
- Array or linked list

Example

```java
class Node {
    Letter value; // Letter V = { V₁, V₂, … Vₘ }
    Node child[ m ];
}
```
Standard Tries

Efficiency

- Uses $O(n)$ space
- Supports search / insert / delete in $O(d \times m)$ time

For:

- $n$: total size of strings indexed by trie
- $d$: length of the parameter string
- $m$: size of the alphabet
Word Matching Trie

- Insert words into trie
- Each leaf stores occurrences of word in the text
Compressed Trie

Observation
- Internal node $v$ of $T$ is redundant if $v$ has one child and is not the root

Approach
- A chain of redundant nodes can be compressed
  - Replace chain with single node
  - Include concatenation of labels from chain

Result
- Internal nodes have at least 2 children
- Some nodes have multiple characters
Compressed Trie

Example
Compact Tries

Compact representation of a compressed trie

Approach

- For an array of strings $S = S[0], \ldots S[s-1]$
- Store ranges of indices at each node
  - Instead of substring
- Represent as a triplet of integers $(i, j, k)$
  - Such that $X = S[i][j..k]$
- Example: $S[0] = \text{“abcd”}$, $(0,1,2) = \text{“bc”}$

Properties

- Uses $O(s)$ space, where $s = \# \text{ of strings in the array}$
- Serves as an auxiliary index structure
Compact Representation

**Example**

- S[0] = see
- S[1] = bear
- S[2] = sell
- S[3] = stock
- S[4] = bull
- S[5] = buy
- S[7] = hear
- S[8] = bell
- S[9] = stop

```
0 1 2 3 4
S[0] = see
S[1] = bear
S[2] = sell
S[3] = stock
S[4] = bull
S[5] = buy
S[7] = hear
S[8] = bell
S[9] = stop
```
**Suffix Trie**

- Compressed trie of all suffixes of text
- Example: “IPDPS”
  - Suffixes
    - IPDPS
    - PDPS
    - DPS
    - PS
    - S
- Useful for finding pattern in any part of text
  - Occurrence $\Rightarrow$ prefix of some suffix
  - Example: find PDP in IPDPS
Suffix Trie

Properties

- For
  - String X with length n
  - Alphabet of size m
  - Pattern P with length d
- Uses $O(n)$ space
- Can be constructed in $O(n)$ time
- Find pattern P in X in $O(d \times m)$ time
  - Proportional to length of pattern, not text
Suffix Trie Example

```
minimize
0 1 2 3 4 5 6 7
```

![Suffix Trie Diagram](image)
Tries and Web Search Engines

- Search engine index
  - Collection of all searchable words
  - Stored in compressed trie

- Each leaf of trie
  - Associated with a word
  - List of pages (URLs) containing that word
    - Called occurrence list

- Trie is kept in memory (fast)

- Occurrence lists kept in external memory
  - Ranked by relevance
Computational Biology

DNA
- Sequence of 4 different nucleotides (ATCG)
- Portions of DNA sequence produce proteins (genes)

Genome
- Master DNA sequence for organism
- For Human
  - 46 chromosomes
  - 3 billion nucleotides
**DNA the molecule of life**

**Trillions of cells**
Each cell:
- 46 human chromosomes
- 2 meters of DNA
- 3 billion DNA subunits (the bases: A, T, C, G)
- Approximately 30,000 genes code for proteins that perform most life functions
Tries and Computational Biology

- ESTs
  - Fragments of expressed DNA
  - Indicator for genes (& location)
  - 5.5 million sequences at NIH

- ESTmapper
  - Build suffix trie of genome
    - 8 hours, 60 Gbytes
  - Search for ESTs in suffix trie
    - 11 hours w/ 8 processor Sun

- Search genome w/ BLAST
  - 5+ years (predicted)