CMSC 330
Homework 2
Due: December 4, 2006

Lambda Calculus Problems

1. Apply β-reduction to each of the following λ-expressions until no more beta reductions can be applied:
   (a) (λ x.xx)(λ y.yx)z
   (b) (λ x. (λ y. (x y)) y) z
   (c) ((λ x.xx)(λ y.y))(λ y.y)
   (d) (((λx. λy. (x y)) (λ y.y)) w)

2. Consider the λ-expression (λ x.y)((λ y. y y y)(λ x. x x x))
   (a) Show that this expression has multiple reduction sequences, some infinite and some finite.
   (b) What is the reduced value of a finite reduction sequence? (All finite reduction sequences have the same result.)

3. Here are three important *combinators*, which are lambda terms with no free variables:

I = λx.x,
K = λx. λy. x
S = λx. λy. λz.(xz(yz)).

I is the identity function, and K is the function of two arguments that ignores its second argument and returns its first. It can be shown that any combinator can be generated from S and K using only function application. Verify that SKK reduces to I.

4. Using the definitions of **true**, **false**, and **and** given in class (see slides), show that **and** behaves correctly as a logical "and" function.

5. We say that f and λx.fx are *extensionally equivalent* if x does not occur free in f. Which of the following λ-expressions are extensionally equivalent, and why or why not?
   (a) (λx.(square x)) and square
   (b) λx.((λy. (f x y)) x) and λy.(f x y)